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Professor H. H. TURNER, M.A., D.Sc, PRESIDENT, in the Chair.

Alfred Pratt, B.A., B.Sc., 14 Endwell Road, Brockley, S.E., was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Commander Philip Dumas, R.N., F.R.G.S., Instructor in Electricity and Submarine Mining, &c., Royal Naval Barracks, Chatham (proposed by E. J. Spitta) ;

Henry Eichbaum, Granville Hill, Eastbourne, Sussex (proposed by G. F. Chambers) ;

Frank Flowers, Map Officer, Ginsberg Chambers, Johannesburg, Transvaal, South Africa (proposed by R. T. A. Innes) ; and

Louis George Macrory, M.D., B.A., &c., Clifton House, Bridge Road, Battersea, S.W. (proposed by A. Fowler).

Seventy-five presents were announced as having been received since the last meeting, including, amongst others :

Photographic map of the heavens ; fifty-five plates taken at the Astronomical Observatory of Harvard College, presented by Professor E. C. Pickering ; a series of photographs (transparencies) of the total solar eclipses of 1898, 1900, and 1901, taken by the Lick Observatory Expeditions, presented by the Lick Observatory ; large transparency spectro-heliograph of the Sun, taken at the Yerkes Observatory, presented by Professor G. E. Hale.

Examination of Mr. Whittaker's "Undulatory Explanation of Gravity" from the Physical Standpoint. By G. Johnstone Stoney, M.A., Hon. Sc.D., F.R.S.

In the part of the *Monthly Notices* issued last September, at p. 619 of vol. lxii., there is a suggestive paper by Mr. Edmund T. Whittaker dealing with three topics.

1. In the first section of his paper he gives a striking and most useful general solution of Laplace's Differential Equation for the potential of gravity, viz. of the equation

$$\nabla^2 V = 0, \text{ or a constant.} \quad \ldots \quad \ldots \quad (1)$$

2. In the second section by a similar method of treatment he obtains a singularly elegant proof of the recently discovered theorem that the wave-motion represented by the equation

$$\nabla^2 V = \frac{1}{k^2} \frac{\partial^2 V}{\partial t^2} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

(in other words, wave-motion which is propagated with constant speed k in all directions through an isotropic medium) can be analysed into components, each of which is an undulation of flat wavelets. This is the principal case of a more general theorem which effects the resolution in crystalline as well as isotropic media; and this proof in the case of isotropic media,* for which we are indebted to Mr. Whittaker, has deduced the resolution from the fundamental differential equation of such motion, and is therefore a most welcome addition to the previously known proofs of the theorem.

3. In the last section of his paper Mr. Whittaker develops "An undulatory explanation of gravity" which aims at "analysing the field of force [? acceleration †] due to a gravitating body by a 'Spectrum Analysis,' as it were, into an infinite number of constituent fields . . . of an undulatory character." This rather

* It seems likely that Mr. Whittaker's remarkable handling of equations (1) and (2) could present the general solution of equation (2) as an integral of elements each of which is an undulation of flat wavelets, in the case where k is a function of the polar coordinates θ and ϕ, as well as in the case dealt with by Mr. Whittaker where k is a constant in reference to θ, ϕ, and r. It would be of much interest to ascertain whether this is so, but the present writer has not gone sufficiently into this question. If so, Mr. Whittaker's analysis will extend to crystalline as well as isotropic media, just as the method of proof by the Principle of Reversal does (see *British Association Report* for 1901, p. 570). It may be desirable to add that the proof by the method of Reversal supplies further information which is of use in making experiments, inasmuch as it exhibits the relation in which the resolution into flat wavelets stands to resolutions into wavelets of other forms. It seems probable that this information could not be obtained without much difficulty by the analytical method.

† See "How the Misuse of the Word *Force* may be avoided," *British Association Report* for 1894, p. 586.

startling announcement seems to require further discussion than that given to it by its author, in order that we may understand what the proposed resolution means *physically* and what it does not mean. The physical meaning of his equations is not referred to in Mr. Whittaker's paper, and an inquiry into it is therefore undertaken here, since without some knowledge about it we are not in a position to form a judgment as to the true value of the resolution.

4. In the department of physics to which Mr. Whittaker's investigation belongs a differential equation has first to be so constructed as to express the fact that physical causes, acting under definite laws, are in operation; and having obtained this differential equation by a study of the laws under which the physical causes act in the problem of nature which is under investigation, then the innumerable particular solutions of that differential equation will represent the several events in nature which those physical causes are competent to produce.

5. Thus, in acoustics, the differential equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 y}{\partial t^2}$$

represents the physical causes that govern the transverse vibrations of a uniform flexible and elastic string when stretched and set vibrating, so long as the range of vibration is small; and the fundamental note emitted by this string and all its harmonics present themselves as motions represented by particular solutions of this equation. In this and all analogous cases the sum of any of these particular solutions will also be a solution of the differential equation; the physical meaning of which is that any group of the harmonics, or of the harmonics and fundamental tone, produce a complex resultant tone of which the members of that group are true physical components.

A similar state of things prevails when we investigate mathematically that spectrum analysis of light to which Mr. Whittaker likens his analysis of gravity. Each of the luminous undulations is a particular solution of the differential equation which is to be employed when dealing with light; and *any* group of these particular solutions when added together represents a resultant luminous effect of which the members of that group are genuine physical components.

The same is also true of that other resolution of light into its flat wavelet components which is dealt with by Mr. Whittaker in the second part of his paper. Here, again, the resultant is the sum of the components, and the components and resultant are all of them solutions of the fundamental differential equation: in other words they are, whether taken separately or as coexisting in any group, effects which the physical causes represented by the differential equation are competent to produce.

The same holds true of fields of potential due to gravitation.

Each attracting centre produces its own field of potential, and the coexistence of any number and any disposition of attracting centres produces a resultant field which is represented mathematically as the sum of the component fields.

6. All these are cases in which, by applying an unrestricted treatment to the differential equation, we obtain true physical resolutions; but the resolution of gravity suggested by Mr. Whittaker is of a different kind. In the first place it does not result from an unrestricted treatment of the differential equation, since it is only by picking out one particular and very special solution of the equation which Mr. Whittaker suggests shall be substituted for Laplace's equation, that we can arrive at his resolution; and it only remains to inquire what is the physical meaning of the process by which this artificial resolution is obtained.

7. To convert Mr. Whittaker's analysis into a form which will exhibit its physical meaning, it will suffice to differentiate his equations by r, and thus transform them from equations that refer to potential into equations that refer to accelerations. The definite case with which Mr. Whittaker deals is that of the field of potential called into existence by an attracting centre situated at the origin of coordinates. In this case his undulatory components assume the form

$$dV = \frac{2d\mu}{\pi\mu} \cdot \frac{\sin \mu(kt-r)}{r} \quad \ldots \quad \ldots \quad (3)$$

where μ is the wave-frequency and k the speed of propagation of the undulation. Differentiating this equation by r we obtain the following expression for the corresponding component of the attracting acceleration:—

$$dR = -\frac{2d\mu}{\pi}\left(\frac{\sin \mu(kt-r)}{\mu} \cdot \frac{1}{r^2} + \cos \mu(kt-r) \cdot \frac{1}{r}\right) \ldots \quad (4)$$

This shows that this component of the field of acceleration consists of two attractions at each distance r from the origin, one an acceleration varying inversely as the distance, the other an acceleration varying inversely as the square of the distance, and both multiplied by coefficients which make their intensity at each point of space fluctuate periodically with the frequency μ.

8. Accordingly the general outcome is that we are to imagine two infinite series of accelerations the terms of which correspond to all possible values of μ, one series consisting of accelerations varying inversely as the distance, the other of accelerations varying inversely as the square of the distance, and all these accelerations flickering in intensity at each point of space in definite ways. Then what the physicist learns from Mr. Whittaker's analysis is that *there is one definite set of coefficients* which if prefixed to these innumerable flickering accelerations will enable them to furnish an unvarying resultant at each point of space.

This is an effect of a kind which it is antecedently probable could by suitable manipulation be brought about; but unfortunately results obtained by such manipulation, though pleasing from their ingenuity, do not hold out any prospect of being of service to the student of dynamical astronomy.

9. Viewed from the mathematical standpoint, Mr. Whittaker's investigation amounts to this, that when we integrate equation (3) we find in it an instance of a state of things occasionally met with—viz. that where a variable, such as r in equation (3), presents itself both under and outside a sign of integration, then this variable as it exists outside the sign of integration will alone survive when the integration is taken between definite limits. This, for example, will happen whenever, as in the present instance, viz., in—

$$V = \frac{2}{\pi r}\int \frac{\sin \mu(kt-r)}{\mu} d\mu \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

or

$$V = \frac{2}{\pi r}\int \frac{\sin \mu(kt-r)}{\mu(kt-r)} d[\mu(kt-r)] \quad \ldots \quad \ldots \quad (3b)$$

the variable in question only appears under the sign of integration in a form in which it can be presented as part of a coefficient to the variable by which we are integrating, and when the limits of integration are then made cipher and infinity. In Mr. Whittaker's integral the transversal (or square root of the intensity) of each element of the series represented by the integral is assumed to be proportional to $1/\mu$, the wave-length, and when this very special set of values are assigned to the coefficients of the terms of the series represented by the integral, then r under the sign of integration, which is essential to the physical interpretation, is excluded from the resultant while appearing in the components. This produces the semblance of a resolution notwithstanding that in the components on the one hand, and in the resultant on the other, we are dealing mathematically with differential equations—equations (1) and (2)—the general solutions of which are not physically connected. What Mr. Whittaker has shown is that one particular solution of equation (2) can be obtained by a definite integration so contrived as to obliterate *in that particular solution* the physical incompatibility which in general prevails between equations (1) and (2). This would, possibly, not be an altogether illegitimate process if the physical events which would justify it were such as we might assume to prevail in actual nature. The physical assumption to be made is that the attraction of each particle of ponderable matter can be of that extraordinary kind described above in paragraphs (7) and (8). But, unfortunately, the improbability of this assumption is so overwhelming that no *physicist* can feel himself justified in seriously arguing from the supposition that it represents the state of things which exists in nature, however

much he may be impressed by the mathematical ingenuity of the investigation.

Postscript.

A discussion followed the reading of this paper in which it was pointed out that there exist motions in the solar system which cannot be wholly accounted for by gravitation acting in accordance with Laplace's Differential Equation; and it was suggested that this indicates that Laplace's equation requires modification, at least for bodies in relative motion. Now if Laplace's equation has to be modified it appears certain from the physical interpretation of Mr. Whittaker's equations that the modification cannot be that substitution of equation (2) for Laplace's equation which is suggested by Mr. Whittaker; and that what we learn from the investigation is that we may limit the inquiry by dismissing this alternative when searching for the modification.

But has it been quite ascertained that the observed motions may not be due to causes already known? We are acquainted with at least three causes acting within the solar system which *must* produce deviations from Newtonian motions. The chief of these is the meteoric streams and sporadic meteorites which dash about in vast swarms within the space through which the visible bodies of the solar system have to travel. Another such cause is the escape of gases from atmospheres (see *Astrophysical Journal* for June 1900, Appendix, p. 369). This escape we have reason to believe takes place chiefly from the side of a planet turned towards the Sun, and therefore produces on the planet a one-sided effect; and a third cause which is known to exist is electric. Modern astronomers are able to compute perturbations with such refinement that it does not seem impossible that the first of the above *veræ causæ* may produce effects of sufficient amount to be detected.

It is noteworthy that at least the first two of these *veræ causæ* would operate with more intensity upon *Mercury* than upon our Earth: witness much other evidence, corroborated by the existence of such phenomena as the Solar Corona and the Zodiacal Light.

Note on the Present Condition of the Lunar Theory.
By E. Nevill.

1. The recent discussion at the March Meeting of the Royal Astronomical Society on Professor Newcomb's note, entitled "On the desirableness of a reinvestigation of the problems growing out of the Mean Motion of the Moon," suggests my furnishing some information with respect to the researches on

the motion of the Moon which have been carried out at the Natal Observatory.

2. First with respect to the comparison of the observed and tabular places of the Moon. The whole mass of observations contained in Professor Newcomb's "Reduction and Discussion of Observations of the Moon before 1750" (Washington, 1878); in Sir G. Airy's "Greenwich Lunar Reductions for 1750 to 1830," and its extension "The Greenwich Lunar Reductions for 1831 to 1851"; and the complete series of Greenwich Observations of the Moon made with the Transit Circle between 1851 and 1899, have all been reduced on a uniform system of star places and elements of reduction, to a homogeneous series of comparison between observation and Hansen's "Tables de la lune."

There are thus obtained a homogeneous complete series of comparison between observations and tables extending over the period 1630 to 1900. This may be called "Data A."

3. The preceding comparison between the observed and tabular places of the Moon is insufficient as it stands to furnish any explanation of the origin of the outstanding discrepancies whose existence it reveals. For this purpose what is required is to so combine the results of the comparison between observation and tables, that they will yield separately for each year—

(1) The coefficient expressing the mean error depending on the Moon's Mean Longitude.

(2) The coefficient expressing the mean errors depending on the sine and cosine of the Moon's *Mean Anomaly*.

(3) The coefficients expressing the mean errors depending on the sine and cosine of the *Variation*.

(4) The coefficients expressing the mean errors depending on the sine and cosine of the *Annual Equation*.

(5) The coefficients expressing the mean errors depending on the sine and cosine of the *Parallactic Inequality*.

(6) The coefficients expressing the mean errors depending on the sine and cosine of the *Evection*.

(7) The coefficients expressing the mean error of the Moon's Parallax and its variation.

(8) The coefficients expressing the mean error varying directly as the Moon's Latitude.

(9) The coefficients expressing the mean errors varying as the sine and cosine of the Argument of Latitude.

The available observations prior to 1750 are too few to enable these to be systematically calculated for each year, but for the one hundred and fifty years from 1750 to 1899, from the preceding comparison between Hansen's tabular place of the Moon and that as observed at Greenwich, there have been computed for each year the value of the preceding eighteen coefficients. In addition the more accurate modern places of the Moon obtained by means of the Greenwich Transit Circle since 1851, have been

utilised for computing the separate yearly values of a number of other coefficients depending on other auxiliary arguments.

The long series of yearly values of these different coefficients furnishes full data for the investigation from the observations of the origin of the outstanding imperfections of Hansen's Tables. They may be called "Data B."

4. These outstanding imperfections in Hansen's Tables may be divided into the following classes :—

(1) Errors in the theoretical values for the coefficients of the perturbations due to the direct action of the Sun employed in the theory embodied in Hansen's "Tables de la lune."

(2) Consequential errors due to Hansen having adopted erroneous values for the different constants depending on observation which are involved in his theory.

(3) Errors in the values assigned by Hansen to the perturbations due to the disturbing action of the planets, the figure of the Earth and similar causes, and omission of terms representing perturbations of sensible magnitude whose existence he had overlooked.

Considering the first of these classes, it may be taken that the values assigned by Hansen in the "Darlegung" to the ordinary lunar solar perturbations correspond very closely to the true theoretical values, if Hansen's values for the constants be adopted. There are few cases where the error in the coefficients exceeds some hundredth of a second of arc. Generally, they agree with the values found by Delaunay, when both are reduced to the same basis. In all cases where there exists any material discrepancy between the theoretical values of Hansen and Delaunay, the correct value of the coefficient has been recomputed, carrying the calculation when necessary to the eleventh power of m. The existing discrepancies arise mainly from Delaunay not having carried his computations far enough to obtain the complete value of the coefficient, but in some cases they are due to Delaunay having overlooked combinations of terms contributing to the complete value. In a very few cases Hansen's value seems incomplete, probably from the accidental omission of terms of high order. It is true that the values of these coefficients embodied in Hansen's Tables are not identical with the final values given in the "Darlegung," but the differences are comparatively unimportant, and no revision of theory will sensibly alter the great mass of theoretical values embodied in Hansen's Tables.

With respect to the second of these classes, the values assigned by Hansen to the constants which have to be derived from observation, nearly all require material amendment. This is especially the case with respect to the secular motion of the *Mean Longitude*, of the *Longitude of Perigee*, and of the *Longitude of Node*, as well as with the value assigned to the *Secular Accelera-*

tion, and the ratio of the parallaxes of the Sun and Moon. But in addition corrections are required to the values employed by Hansen for the constants of *Eccentricity* of *Inclination* and of *Parallax.*

The corrections to Hansen's tabular values of these constants have been derived from the complete series of values termed "Data B," and Hansen's Tables amended by supplying the requisite auxiliary tables necessary to take them into account. For the sake of completeness it has been necessary to do this on two separate systems.

1st. By supposing that Hansen's Tables are complete as they stand, and that after rectifying an admitted error, they merely require the removal of the admittedly incorrect empirical term depending on the argument (8 V − 13 E).

2nd. By admitting that Hansen's Tables are not complete, but after the removal of the empirical term, require supplementing by the addition of certain smaller terms due to the action of the planets, such as the *Jovian Evection*, of whose existence and approximate value there can be no question.

Yet the difference between the two resulting systems of corrections is far less than might have been imagined.

It is with the third class of imperfections in Hansen's Tables that the main difficulty arises. The values which Hansen determined from theory for the coefficients of the perturbations of this character, all need some correction, for in no case has he carried his computations far enough to obtain the complete value. Generally speaking, the difference is not large enough to be of importance, but in the case of the terms depending on the longitude of the Moon's Node, and the perturbation of *Venus*, there is a considerable difference between Hansen's value and the true values of the terms.

But the crucial point is: How many terms of this class having sensible magnitude are absent from Hansen's Tables? About half a dozen there can be no question, mainly the planetary evectional terms; but how many beyond these, and of what magnitude and what period? Therein lies the difficulty. Opinion varies.

5. The long series of values for each year of the coefficients of the principal inequalities in the motion of the Moon, which constitutes "Data B," can be made to show what are the inequalities which are not included in Hansen's Tables, and yet are necessary to reconcile the observed and tabular places of the Moon.

They have been employed for this purpose, and by the suitable discussion of the variations in the yearly values of the twenty different coefficients, there have been deduced the probable periods of the different outstanding terms, and the approximate values of their coefficients.

Some of these outstanding terms are found to correspond closely in both period and coefficient with those known to exist and to be absent from Hansen's Tables, and the fact that the values so determined correspond so closely with the values furnished by theory furnishes the strongest evidence of the reality of the existence of the other similar terms whose values are not so readily determined from theory.

6. The full details of the results of the reduction and discussion of this great mass of observations as compared with Hansen's Tables, has been lying for some time at the Natal Observatory awaiting publication. For it was decided by the Natal Government that as the work represented a considerable part of the scientific work done at the Colonial Observatory, it must not be published in any other form than as part of the publications of the colony. Up to the present no provision has been made to meet the cost of printing, but in the estimates for the coming year, it is trusted, provision will be made for the due printing and publication of this investigation. If so, they will appear in the course of the coming year, and the enormous mass of work be available for general use.

7. With regard to the theoretical calculation of these inequalities it is not necessary to say anything, as my views are well known. Most of those which require consideration have been calculated by two independent methods which have been already described in the *Monthly Notices* of the Society. So far the final reduction and numerical calculation has been suspended, so that the values deduced from the observations shall remain absolutely independent. The theoretical determination of these terms will form, therefore, a subsequent volume of the publications of the Natal Observatory as soon as the necessary funds for printing are available; and as it will be a heavy mass of mathematical printing, its cost will be much heavier than the portion of the complete whole formed by the reduction and discussion of the Greenwich lunar observations of the last 150 years.

Natal Observatory:
1903 *May* 9.

Note on the use of Peirce's Criterion for the Rejection of Doubtful Observations. By S. A. Saunder, M.A.

It would seem at first sight that when for observations made at the telescope we substitute a series of photographic measures we ought to be able to eliminate all those abnormal errors or real mistakes, such as the entry of a wrong figure, which are not con-

templated in the Theory of Errors, but which undoubtedly do occur in practice, and which have led to the suggestion of various criteria for the detection and consequent rejection of such abnormal observations.

With regard to such classes of mistakes as may be supposed to have most frequently occurred in the case of telescopic observations, this is undoubtedly the case, for a photographic measure which gives an abnormal result, and so comes under suspicion, may be repeated any number of times, and any clerical error or mistake in reading may be certainly detected and eliminated.

But a new possibility of error is introduced at the same time. A plate is either accepted or rejected as a whole in accordance with the general excellence or the reverse of its definition. But it may well happen that a plate which gives excellent definition as a whole may yet have some defective images. How far this is likely to occur in star photographs, such as those taken for the astrographic catalogue, I have no experience ; but it does, and I think must, happen in every photograph of the Moon. Formations near the terminator may be beautifully sharp, whilst from unsuitable illumination, or from the general brightness of a district, those in other parts may be quite unmeasurable. Between these two extremes there will be an almost continuous gradation, and it becomes an anxious question where to draw the line between those formations which are to be measured and those to be rejected. Another possible cause of error may be that the assumed positions of some of the standard points are seriously wrong. Where these standard points are stars which have been repeatedly observed with meridian instruments such errors are not likely to be great ; but in an investigation such as a survey of the Moon, where in our present state of knowledge the standard points have to be built up as we proceed, it is a possibility not to be lost sight of. There is too the, probably small, risk of distortion of the film.

In a recent difficulty, due to an attempt to measure two badly defined formations, I found Peirce's criterion so useful that I thought it might be of interest to call attention to it, especially as I believe that the criterion is seldom if ever employed by the most experienced computers, at all events in this country.

The plate in question was one of those most kindly placed at my disposal by M. Loewy ; it was taken at the Paris Observatory on 1899, March 22, and shows something like three quarters of the disc under morning illumination with a terminator at about $-41°$. Some 300 or 400 points have been measured by Mr. Hardcastle on this plate with considerable certainty, but most of them are near the terminator, and in order to determine the constants of the plate it was very desirable to have some points near the west limb. Two of the best determined of such points are *Macrobius a* and *Proclus*, whose positions have been measured 11 and 12 times respectively by Dr. Franz with the Königsberg

heliometer.* The formations were both over exposed on the plate, and Mr. Hardcastle noted at the time he made the measures that considerable uncertainty attached to them. The value of such points is, however, so great for determining the scale and orientation that I decided to include them in a group of 45 points, giving 90 conditional equations, from which the four constants of the plate were to be determined.

When the solution was completed it was found that *Macrobius a* gave a residual of 65 in ξ, *Proclus* one of 68 in η; the unit being ·00001 of the Moon's radius, so that each residual was about 0″·6. These were the greatest residuals found; they were so large as to make me uneasy about them, and yet not so large that I could unhesitatingly say that they ought to be rejected. In this dilemma I computed the value of Peirce's criterion, which gave 63 as the limit for rejection of a single observation, the mean error of a co-ordinate being 22·9. The critical value for a third observation was 54, and the next residual actually occurring was 50. The criterion thus gave me just the additional assistance I required in order to come to a decision; I rejected the two points and solved again. The mean error fell to 20·1, the value of the Moon's radius, giving the scale of the plate, was altered by 0″·06 with a probable error $\pm$ 0″·043, the orientation was altered by 33″ with a probable error $\pm$ 9″·7, the discordances in the positions of the two rejected points each rose to 74, and the residual of 50 fell to 46. The general impression left on my mind was that the solution was much improved.

The general question of the rejection of discordant observations has been discussed by Dr. Glaisher,† who strongly upholds De Morgan's view that when recognisable mistakes have been eliminated no observation should be rejected, but that when the first solution has been effected the conditional equations should be re-weighted in accordance with their residuals and solved again, the process being continued until the weights assumed for the last solution are sufficiently near to those given by that solution. How many solutions this would in general require I do not know, but it is clear that the process might be a very laborious one. In many cases its length would be practically prohibitive, and if we decide to give all the equations, either unit or zero weight, it is certain, as Dr. Glaisher points out, that we shall get a better result by excluding certain discordant observations than we should by including them.

I believe that the practice amongst computers of experience is to rely almost entirely on their individual judgment, taking into account the conditions of the observations, and drawing the line somewhere about those observations which give residuals of five times the probable error. But for those who, like myself, have a very limited practical acquaintance with least square

* *Königsberg Beobach.*, Band 38; 'Die Fig. des Mondes,' pp. 10, 11.
† *Monthly Notices*, Vol. xxxiii., pp. 391–402.

solutions, difficult cases, like the one I have just quoted, must from time to time arise, and I cannot help thinking that others may be as glad as I have been to have recourse to a criterion established on some definite principle, which may be considered as an embodiment of that accumulated experience which we lack.

The computation of the criterion is quite a short matter with the help of such tables as those given in Chauvenet, but it is only to be effected through successive approximations by trial and error, and in order to facilitate its application I have computed a short table which I append to this paper.

If ε denote the mean error (error of mean square) as given by the least square solution, the critical residual may be denoted by $\kappa\varepsilon$, where κ is the quantity it was the object of Professor Peirce's investigation to discover. Values of κ^2 are given in Chauvenet, as computed by Gould, for any number of conditional equations up to 60 involving either one or two unknowns. But in determining the constants of a photographic plate the number of unknowns which enter into a group of conditional equations is usually either three or four. I have accordingly computed the values of κ for these cases and for numbers of conditional equations between 10 and 100.

It did not seem necessary to make the table as elaborate as Dr. Gould's, since interpolation is easy, and the values given are, even theoretically, only approximate. The theoretical value depends upon the ratio of the two values of ε when the questionable observations are included, and when they are excluded ; this ratio must vary in different cases, but an approximation is made by assuming that the sum of the squares of the residuals is diminished only by the squares of those given by the observations rejected. This gives the new value of ε a little too large and consequently makes κ a little too small. But as we are in the neighbourhood of a minimum value of the sum of the squares the difference is usually small.

Also it must be remembered that the criterion is to be considered only as a guide to the computer on whose judgment the final decision must ultimately depend.

In the following table

m denotes the number of conditional equations ;

μ the number of unknown quantities ;

n the number of observations it is proposed to reject.

It must be particularly borne in mind that the criterion for $n = 2$ is not to be applied unless one observation has been rejected by that for $n = 1$; that for $n = 3$, not unless two have been rejected by that for $n = 2$, and so on.

Peirce's Criterion.

Values of κ.

	$\mu = 3$.					$\mu = 4$.			
m	n				m	n			
	1	2	3	4		1	2	3	4
10	1·72	1·44	1·28	1·16	10	1·63	1·37	1·22	1·11
12	1·83	1·55	1·38	1·26	12	1·76	1·49	1·33	1·22
14	1·92	1·64	1·47	1·35	14	1·86	1·59	1·43	1·31
16	2·00	1·72	1·55	1·42	16	1·94	1·67	1·50	1·38
18	2·06	1·78	1·61	1·48	18	2·01	1·74	1·57	1·45
20	2·12	1·84	1·67	1·54	20	2·07	1·80	1·63	1·51
25	2·23	1·96	1·79	1·66	25	2·19	1·92	1·76	1·63
30	2·32	2·05	1·88	1·75	30	2·29	2·02	1·85	1·73
35	2·39	2·12	1·95	1·83	35	2·36	2·10	1·93	1·81
40	2·45	2·19	2·02	1·90	40	2·43	2·16	2·00	1·88
45	2·51	2·24	2·08	1·95	45	2·48	2·22	2·06	1·94
50	2·55	2·29	2·13	2·00	50	2·53	2·27	2·11	1·99
60	2·63	2·37	2·21	2·09	60	2·61	2·35	2·19	2·08
70	2·69	2·44	2·28	2·16	70	2·67	2·42	2·26	2·15
80	2·74	2·49	2·34	2·22	80	2·73	2·48	2·32	2·21
90	2·79	2·54	2·39	2·27	90	2·77	2·53	2·37	2·26
100	2·83	2·58	2·43	2·32	100	2·81	2·57	2·42	2·31

On Oscillating Satellites. By H. C. Plummer, M.A.

1. In his paper "On Periodic Orbits" * (to be quoted here as *Ch.*) Professor Charlier has tried to obtain in a simple analytical manner some of the results already found by Professor Darwin in his well-known memoir † on this subject (to be quoted as *D.*). The orbits investigated by Professor Charlier are those of oscillating satellites—*i.e.* those small periodic orbits which can be described in the neighbourhood of the five points where exact solutions of the problem of three bodies are known to exist. Some of his results I have tried to discuss in a more general way in a paper ‡ (to be quoted as *Pl.*) in which I have considered the motion of a small body in the neighbourhood of centres of libration, or points of zero force, in any rotating field of force. In the present paper it is sought to push the analysis beyond the

* *Meddelanden från Lunds Astronomiska Observatorium*, No. 18.
† *Acta Mathematica*, t. xxi. p. 99.
‡ *Monthly Notices*, vol. lxii. p. 6.

first degree of approximation previously obtained and to consider more particularly the orbits in the neighbourhood of the three collinear points of libration, especially those of families a and b, for which numerical results are to be found in *D*.

2. As far as possible the notation of *Pl.* will be retained. The equations of motion of the third mass in the most restricted form of the problem of three bodies, referred to axes rotating with the line joining the finite masses, are

$$\ddot{x}-2\dot{y}=\frac{\partial\Omega}{\partial x},\quad \ddot{y}+2\dot{x}=\frac{\partial\Omega}{\partial y}$$

where

$$2\Omega=\mu\left(r_1^2+\frac{2}{r_1}\right)+\nu\left(r_2^2+\frac{2}{r_2}\right)$$

The masses of the finite bodies are μ and ν, and the unit of mass is such that $\mu+\nu=1$. The origin is taken at the centre of gravity of the two revolving bodies, these lying on the axis of x and their distance apart being the unit of length. The unit of time is such that the angular velocity of the axes is also unity. The partial differentials of Ω with respect to x and y are: of the first order, p and q; of the second order, r, s, and t *; of the third order, P, Q, R, and S. Then if the point (x,y) is a centre of libration, for which $p=q=0$, and the coordinates of the small mass are $x+\xi$, $y+\eta$, the equations of motion may be written

$$\ddot{\xi}-2\dot{\eta}=r\xi+s\eta+\tfrac{1}{2}(\mathrm{P}\xi^2+2\mathrm{Q}\xi\eta+\mathrm{R}\eta^2)+\ \ldots$$
$$\ddot{\eta}+2\dot{\xi}=s\xi+t\eta+\tfrac{1}{2}(\mathrm{Q}\xi^2+2\mathrm{R}\xi\eta+\mathrm{S}\eta^2)+\ \ldots$$

so long as the body continues within the region where the expansions are legitimate. To the first order of the relative coordinates the solution is already known. It is

$$\xi=h\cos(mt-\alpha),\quad \eta=k\cos(mt-\beta)$$

where

$$m^4+(r+t-4)\,m^2+rt-s^2=0$$
$$h^2=a^2(t+m^2),\quad k^2=a^2(r+m^2)$$
$$\tan(\alpha-\beta)=-2m/s$$

These equations, found in *Pl.*, §§ 4 and 5†, leave the origin of time, depending on α, and the scale of the orbit, depending on a, arbitrary.

3. To proceed to the second approximation we may put

$$\xi=h\cos(mt-\alpha)+\xi',\qquad \eta=k\cos(mt-\beta)+\eta'$$

* The symbol t also represents the time; but this can hardly cause any confusion.

† The constant a used above is connected with the constant c used in *Pl.* by the relation $a^2(4m^2+s^2)=c^2$.

where h, k are small quantities of the first order, ξ', η' of the second order. In virtue of the preceding relations among the constants, the equations of motion now become

$$\left.\begin{aligned} \ddot{\xi}'-2\dot{\eta}' &= r\xi'+s\eta'+\mathrm{U} \\ \ddot{\eta}'+2\dot{\xi}' &= s\xi'+t\eta'+\mathrm{V} \end{aligned}\right\} \quad \text{(A)}$$

where if terms beyond the second order be neglected

$$2\mathrm{U}=\mathrm{P}h^2\cos^2(mt-\alpha)+2\mathrm{Q}hk\cos(mt-\alpha)\cos(mt-\beta)+\mathrm{R}k^2\cos^2(mt-\beta)$$

$$2\mathrm{V}=\mathrm{Q}h^2\cos^2(mt-\alpha)+2\mathrm{R}hk\cos(mt-\alpha)\cos(mt-\beta)+\mathrm{S}k^2\cos^2(mt-\beta)$$

Hence

$$\mathrm{U}=a_1+b_1\cos 2mt+c_1\sin 2mt$$
$$\mathrm{V}=a_2+b_2\cos 2mt+c_2\sin 2mt$$

where

$$4a_1=\mathrm{P}h^2+2\mathrm{Q}hk\cos(\alpha-\beta)+\mathrm{R}k^2$$
$$4b_1=\mathrm{P}h^2\cos 2\alpha+2\mathrm{Q}hk\cos(\alpha+\beta)+\mathrm{R}k^2\cos 2\beta$$
$$4c_1=\mathrm{P}h^2\sin 2\alpha+2\mathrm{Q}hk\sin(\alpha+\beta)+\mathrm{R}k^2\sin 2\beta$$

and the corresponding equations for a_2, b_2, and c_2 are found by substituting Q, R, S for P, Q, R respectively. Then the equations (A) will be satisfied by the solution

$$\xi'=f_1+h_1\cos 2mt+k_1\sin 2mt$$
$$\eta'=f_2+h_2\cos 2mt+k_2\sin 2mt$$

provided that

$$rf_1+sf_2+a_1=0$$
$$sf_1+tf_2+a_2=0$$

and that

$$\begin{aligned} &(4m^2+r)h_1+sh_2 &&+4mk_2 &&+b_1=0 \\ &-4mh_2 &&+(4m^2+r)k_1+sk_2 &&+c_1=0 \\ &sh_1 &&+(4m^2+t)h_2-4mk_1 &&+b_2=0 \\ &4mh_1 &&+sk_1 &&+(4m^2+t)k_2+c_2=0 \end{aligned}$$

From this latter set of four equations we obtain

$$\begin{aligned} \delta\,.\,h_1 &= -(4m^2+t)b_1+sb_2 &&+4mc_2 \\ \delta\,.\,h_2 &= sb_1 &&-(4m^2+r)b_2-4mc_1 \\ \delta\,.\,k_1 &= &&-4mb_2 &&-(4m^2+t)c_1+sc_2 \\ \delta\,.\,k_2 &= 4mb_1 &&+sc_1 &&-(4m^2+r)c_2 \end{aligned}$$

where

$$\delta=3(4m^4+s^2-rt)$$

4. From these results obtained by proceeding to the second order of the dimensions of the relative orbit we have two general properties which characterise the orbits of oscillating satellites :

(1) The orbits of any family are tautochronous to this order of approximation.

(2) The mean position of the small body is displaced by an amount of the second order from the corresponding centre of libration.

In this way it should be possible to find, with little extra trouble, better approximations to the relative paths of oscillating satellites than the elliptic orbits discussed in *Ch.* The results given in *D.* for satellites of families *a* and *b* show that tautochronism ceases to hold at a comparatively small distance from a centre of libration. This fact restricts the application of (1) to very limited areas, and suggests that an independent study of the variation of the period in this class of orbit would be very valuable.

5. The formulæ are much simplified when the collinear centres of libration in the special problem are considered. In this case $y=0$, and we have for the partial differentials of Ω

$$p=\pm\mu\cdot\left(r_1-\frac{1}{r_1^2}\right)\pm\nu\cdot\left(r_2-\frac{1}{r_2^2}\right)=0$$

$$q=0$$

$$r=\mu\left(1+\frac{2}{r_1^3}\right)+\nu\left(1+\frac{2}{r_2^3}\right)$$

$$s=0$$

$$t=\mu\left(1-\frac{1}{r_1^3}\right)+\nu\left(1-\frac{1}{r_2^3}\right)$$

$$\mathrm{P}=\pm\mu\cdot\left(-\frac{6}{r_1^4}\right)\pm\nu\cdot\left(-\frac{6}{r_2^4}\right)$$

$$\mathrm{Q}=0$$

$$\mathrm{R}=-\tfrac{1}{2}\mathrm{P}$$

$$\mathrm{S}=0$$

The rule for the ambiguous signs is this: For the orbits between the finite masses (family *a*) we take $+\mu$, $-\nu$; for the orbits beyond ν (family *b*) we take $+\mu$, $+\nu$; for the orbits beyond μ (family *c*) we take $-\mu$, $-\nu$. Now for comparison with *D.*, $\mu=\frac{10}{11}$, $\nu=\frac{1}{11}$; and taking the values of r_1 and r_2 from *D.*, pp. 108–111, we obtain:

	Family *a*.	Family *b*.	Family *c*.
r_1	0·71751	1·34700	0·94693
r_2	0·28249	0·34700	1·94693
r	13 9878	6·0955	3·1660
t	− 5·4939	− 1·5478	−0·0830
P	+65·076	−39·279	+6·822
R	−32·538	+19·639	−3·411
m	2 6082	1·6763	1·0706
m^2	6·8025	2·8099	1·1463

6. Further simplifications are effected by making $\alpha=0$, $\beta=\frac{1}{2}\pi$. Then

$$4a_1 = Ph^2+Rk^2 = P(h^2-\tfrac{1}{2}k^2) = 4a^2a_1$$
$$4b_1 = Ph^2-Rk^2 = P(h^2+\tfrac{1}{2}k^2) = 4a^2\delta\,.\,a_2$$
$$4c_2 = 2Rhk = -Phk = 4a^2\delta\,.\,a_3$$
$$c_1 = a_2 = b_2 = 0$$

and hence

$$f_1 = -a^2a_1/r$$
$$h_1 = -(4m^2+t)a^2a_2+4ma^2a_3$$
$$k_2 = 4ma^2a_2-(4m^2+r)a^2a_3$$
$$f_2 = h_2 = k_1 = 0$$

The following numerical values result :—

	Family *a*.	Family *b*.	Family *c*.
log δ	2·89534	2·09008	1·21887
log a_1	$2{\cdot}16976_n$	1·49597	$0{\cdot}27043_n$
log a_2	9·38435	$9{\cdot}65902_n$	9·52077
log a_3	9·03335	$9{\cdot}42739_n$	9·34366
log f_1/a^2	1·02401	$0{\cdot}71096_n$	9·76992
h_1/a^2	−4·1352	+2·6261	−0·5487
k_2/a^2	−1·9209	+1·5800	−0·2895
log h/a	$0{\cdot}05840_n$	$0{\cdot}05055_n$	$0{\cdot}01333_n$
log k/a	0·65893	0·47482	0·31736

The coefficients h and k are taken of opposite sign in accordance with the equation (*Pl.* p. 9).

$$(r+m^2)h \sin(\alpha-\beta) = 2mk$$

7. Thus the equations of the three families of orbits are obtained. They are, for family *a* :

$$\xi = +10{\cdot}5685\,a^2-1{\cdot}1439\,a\cos 2{\cdot}6082t-4{\cdot}1352\,a^2\cos 5{\cdot}2164t$$
$$\eta = \qquad +4{\cdot}5597\,a\sin 2{\cdot}6082t-1{\cdot}9209\,a^2\sin 5{\cdot}2164t\,;$$

for family *b* :

$$\xi = -\ 5{\cdot}1400\,a^2-1{\cdot}1234\,a\cos 1{\cdot}6763t+2{\cdot}6261\,a^2\cos 3{\cdot}3526t$$
$$\eta = \qquad +2{\cdot}9841\,a\sin 1{\cdot}6763t+1{\cdot}5800\,a^2\sin 3{\cdot}3526t\,;$$

and for family *c* :

$$\xi = +\ 0{\cdot}5887\,a^2-1{\cdot}0312\,a\cos 1{\cdot}0706t-0{\cdot}5487\,a^2\cos 2{\cdot}1412t$$
$$\eta = \qquad +2{\cdot}0766\,a\sin 1{\cdot}0706t-0{\cdot}2895\,a^2\sin 2{\cdot}1412t$$

8. The next step is to connect the parameter a with the con-

stant C of the Jacobian integral $V^2 = 2\Omega - C$. The equations representing the orbits being of the form

$$\xi = h' \cos mt \,.\, a + (f_1' + h_1' \cos 2mt) \,.\, a^2$$
$$\eta = k' \sin mt \,.\, a + \quad k_2' \sin 2mt \,.\, a^2$$

we have at the time $t = 0$,

$$\xi_0 = h' \,.\, a + (f_1' + h_1')\, a^2 \,;\ \eta_0 = 0$$
$$\dot{\xi}_0 = 0 \,;\ \dot{\eta}_0 = mk' \,.\, a + 2mk_2' \,.\, a^2$$

Hence to the third order in a,

$$V^2 = m^2 k'^2 \,.\, a^2 + 4m^2 k' k_2' \,.\, a^3$$
$$2\Omega = C_0 + r\xi_0^2 + \tfrac{1}{3} P\xi_0^3$$
$$= C_0 + rh'^2 \,.\, a^2 + \{2rh'(f_1' + h_1') + \tfrac{1}{3} Ph'^3\} \,.\, a^3$$

where C_0 is the value of 2Ω at the point of libration. Therefore

$$C - C_0 = (rh'^2 - m^2 k'^2) a^2 + \{2rh'(f_1' + h_1') - 4m^2 k' k_2' + \tfrac{1}{3} Ph'^3\} a^3$$

Now the coefficient of a^3 herein vanishes, as can be proved without difficulty. A useful check on the accuracy of the numbers occurring in § 7 is thus obtained, and the last equation can therefore be written in the simple form

$$C_0 - C = (m^4 - rt)\, a^2$$

which is equivalent to a formula in *Pl.* (§ 7). The numbers for the three cases are these :

	(*a*)	(*b*)	(*c*)
C_0	3·65292	3·53418	3·17322
$m^4 - rt$	123·13	17·33	1·576

9. That the coefficient of a^3 in the expansion of $C - C_0$ must vanish is clear from the fact that its sign would be changed by taking the initial conditions corresponding to $mt = \pi$. And the verification is easily obtained from the general relations of § 3 as simplified in §§ 5 and 6. For we have

$$\frac{h'}{k'} = \frac{-2m}{r + m^2} = \frac{t + m^2}{-2m} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$
$$4rf_1' + Ph'^2 + Rk'^2 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$
$$(4m^2 + r)\, h_1' + 4mk_2' + \tfrac{1}{4}(Ph'^2 - Rk'^2) = 0 \quad \ldots \quad (3)$$
$$4mh_1' + (4m^2 + t)\, k_2' + \tfrac{1}{2} Rh'k' = 0 \quad \ldots \quad \ldots \quad (4)$$

Let (3) be multiplied by m and (4) by $(m^2 + r)$. The difference, when account is taken of (1), becomes

$$3mrh_1' + 3(m^2 + r)\, m^2 k_2' - \tfrac{1}{4} m (Ph'^2 + 3Rk'^2) = 0$$

which again becomes when multiplied by $2h'/3m$

$$2rh'h_1' - 4m^2k'k_2' - \tfrac{1}{6}h'\,(Ph'^2 + 3Rk'^2) = 0$$

Hence by (2)

$$2rh'(f_1' + h_1') - 4m^2k'k_2' + \tfrac{1}{3}Ph'^3 = 0$$

which is the result required.

10. Four examples of orbits of family a are given in *D.* (pp. 233–238), the values of 11 C being 40·0, 39·0, 38·5, and 38·0. These give the numbers

$$11\,(C_0 - C) = 0{\cdot}1821,\quad 1{\cdot}1821,\quad 1{\cdot}6821,\quad 2{\cdot}1821$$
$$\log a = 8{\cdot}06428,\ 8{\cdot}47046,\ 8{\cdot}54705,\ 8{\cdot}60356$$

Whence the following four sets of equations for the orbits result :

$$(\alpha)\quad \xi = +{\cdot}0014 - {\cdot}0133\cos mt - {\cdot}0006\cos 2mt$$
$$\eta = \quad\quad + {\cdot}0529\sin mt - {\cdot}0003\sin 2mt$$
$$(\beta)\quad \xi = +{\cdot}0092 - {\cdot}0338\cos mt - {\cdot}0036\cos 2mt$$
$$\eta = \quad\quad + {\cdot}1347\sin mt - {\cdot}0017\sin 2mt$$
$$(\gamma)\quad \xi = +{\cdot}0131 - {\cdot}0403\cos mt - {\cdot}0051\cos 2mt$$
$$\eta = \quad\quad + {\cdot}1607\sin mt - {\cdot}0024\sin 2mt$$
$$(\delta)\quad \xi = +{\cdot}0170 - {\cdot}0459\cos mt - {\cdot}0067\cos 2mt$$
$$\eta = \quad\quad + {\cdot}1830\sin mt - {\cdot}0031\sin 2mt$$

For comparison with Professor Darwin's calculations the following numbers may suffice. They give the points where the orbits cross the axis of x and the coordinates of the points for which $mt = \frac{1}{2}\tau$; for the latter the ordinates corresponding to the computed abscissæ are found by interpolation among the coordinates given in *D.*

$mt =$	0	π		$\frac{1}{2}\pi$	
	ξ_0	ξ_1	$\xi_0+\xi_1$	ξ'	η'
(α)	−·0125	+·0141	+·0016	+·0020	+·0529
D.	−·0125	+·0147	+·0022	,,	+·0530
(β)	−·0282	+·0394	+·0112	+·0128	+·1347
D.	−·0304	+·0413	+·0109	,,	+·1305
(γ)	−·0323	+·0483	+·0160	+·0182	+·1607
D.	−·0361	+·0523	+·0162	,,	+·1691
(δ)	−·0356	+·0562	+·0206	+·0237	+·1830
D.	−·041	+·060	+·019	,,	+·197

In every case ξ' is slightly less than the abscissa corresponding to the maximum ordinate in *D.*, as should be the case.

11. Two examples of orbits of family *b* are given in *D.* (pp. 239–242), the values of 11 C being 38·5 and 38·0. These give

$$\begin{aligned} 11\ (C_0 - C) &= 0{\cdot}3760, & 0{\cdot}8760 \\ \log a &= 8{\cdot}64750, & 8{\cdot}83116 \end{aligned}$$

and the two corresponding pairs of equations for the orbits are

$$\begin{aligned} (\alpha)\quad \xi &= -{\cdot}0101 - {\cdot}0499\cos mt + {\cdot}0052\cos 2mt \\ \eta &= \quad\ + {\cdot}1325\sin mt + {\cdot}0031\sin 2mt \\ (\beta)\quad \xi &= -{\cdot}0236 - {\cdot}0762\cos mt + {\cdot}0121\cos 2mt \\ \eta &= \quad\ + {\cdot}2023\sin mt + {\cdot}0073\sin 2mt \end{aligned}$$

The results of a comparison with *D.* are given in a form similar to that used in the preceding paragraph.

$mt =$	0	π		$\frac{1}{2}\pi$	
	ξ_0	ξ_1	$\xi_0+\xi_1$	ξ'	η'
(α)	−·0548	+·0450	−·0098	−·0153	+·1325
D.	−·0551	+·0445	−·0106	,,	+·1312
(β)	−·0877	+·0647	−·0230	−·0357	+·2023
D.	−·0875	+·0679	−·0196	,,	+·1987

In both cases ξ' is slightly greater than the abscissa corresponding to the maximum ordinate in *D.*, as the above equations require.

12. Oscillating satellites belonging to the family *c* have not been investigated by Professor Darwin, and there seems to be no means of judging the accuracy of the above results for this family. The comparisons contained in §§ 10 and 11 for satellites of the families *a* and *b* seem to show a fairly good agreement, especially, as was to be expected, in the case of the smaller orbits. For the larger orbits, particularly those of family *a*, the results are by no means so good; and it may be noted that Professor Darwin himself found the orbit (δ) "exceedingly troublesome." But by going to a second approximation it is clear that a better notion has been obtained both of the mean position and shape of the orbit than was possible with the elliptic paths investigated by Professor Charlier. The results found in *Ch.* (*e.g.* p. 25) are really better than could fairly have been anticipated owing to the fact, mentioned in § 8, that the coefficient of a^3 in the development of C vanishes. Further progress in the study of the particular problem considered in the present paper seems to depend, as noticed in § 4, on a knowledge of the variation of the period and the effect of that variation on the form of the orbit.

University Observatory, Oxford:
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Expressions correctes de l'Heure et des Coordonnées des Etoiles dans le système de l'Axe instantané. Par F. Folie, Liège.

(*Communicated by Professor G. H. Darwin.*)

1. Dans le *Bulletin de l'Académie royale de Belgique* (janvier 1903) M. Darwin a démontré que, si l'on fait abstraction des forces perturbatrices, l'expression de l'heure, dans le système de l'axe instantané, renferme un terme de période eulérienne (305 j. pour une terre rigide) qui a pour coefficient $0^s{\cdot}01$ multiplié par la tangente de la latitude ; en sorte que, le même jour et au même instant, les heures déterminées en deux observatoires situés sur un même méridien à $+45°$ et à $-45°$ de latitude diffèrent entre elles de $0^s{\cdot}02$.

J'avais pensé que cette raison suffirait pour faire rejeter le système de l'axe instantané.

Mais de bons astronomes persistent à croire qu'il n'y a néanmoins pas lieu de renoncer à ce système.

Je me propose de démontrer qu'il expose à des erreurs telles qu'on doit nécessairement l'abandonner.

2. Pour cela je devrais rechercher les expressions des coordonnées rapportées à l'axe instantané dans le cas de la nature, c'est-à-dire en tenant compte exactement des forces perturbatrices.

Je me bornerai toutefois à envisager le principal effet de ces forces, la précession, puisqu'il est le seul auquel il soit nécessaire d'avoir égard en pratique.

Soient p, q, r les vitesses angulaires de la terre autour des trois axes principaux X, Y, Z ; α', β', γ' les angles de l'axe instantané avec ceux-ci, $\omega=\sqrt{p^2+q^2+r^2}$.

I. On a $\cos\alpha'=\frac{p}{\omega}$, $\cos\beta'=\frac{q}{\omega}$, $\cos\gamma'=\frac{r}{\omega}$, et $p=\gamma_1\cos\Gamma+c_1\sin\phi$, $q=\gamma_1\sin\Gamma+c_1\cos\phi$, $r=n=c^{te}$. La période de $\Gamma=n\mu t-\sigma$ est de 305 j. pour une Terre rigide.

Des équations précédentes on tire

$$\sin^2\gamma'=\frac{p^2+q^2}{n^2+p^2+q^2},$$ où le dénominateur peut se réduire à n^2 ;

d'où, en faisant $\frac{\gamma_1}{n}=\gamma_1$, $\frac{c_1}{n}=c$:

II. $\sin^2\gamma'=\gamma^2+c^2+2\gamma c\sin(\Gamma+\phi)$ et $\sin\gamma'=\gamma+c\sin(\Gamma+\phi)$.

Désignons par Γ' l'angle que la projection de l'axe instantané fait avec l'axe des X. On a

$$\cos\alpha'=\sin\gamma'\cos\Gamma',\quad \cos\beta'=\sin\gamma'\sin\Gamma'.$$

En nous bornant aux termes du premier ordre, nous pourrons écrire n au lieu de ω dans les expressions de $\cos\alpha'$ et de $\cos\beta'$, et γ' au lieu de $\sin\gamma'$. Alors

$$\gamma'\cos\Gamma'=\gamma\cos\Gamma+c\sin\phi,$$
$$\gamma'\sin\Gamma'=\gamma\sin\Gamma+c\cos\phi,$$

d'où l'on déduit aisément

III. $\Gamma'=\Gamma-c\cos(\phi+\Gamma)$, $\gamma'=\gamma+c\sin(\phi+\Gamma)$.

Abstraction faite de la précession, γ' se réduit à γ, Γ' à Γ.

3. De $$\frac{d\theta}{dt}=-p\cos\phi+q\sin\phi.$$

$$\sin\theta\frac{d\psi}{dt}=p\sin\phi+q\cos\phi, \text{ on tire}$$

$$\frac{d\theta}{dt}=-\gamma_1\cos(\Gamma+\phi) \text{ et } \theta=\theta_0-\frac{\gamma}{1+\mu}\sin(\Gamma+\phi).$$

$$\sin\theta\frac{d\psi}{dt}=\gamma_1\sin(\Gamma+\phi)+c_1 ; \sin\theta(\psi-\psi_0)=c_1t-\frac{\gamma}{1+\mu}\cos(\Gamma+\phi).$$

Si l'on considère le triangle EE_1F, dans lequel les arcs EF, E_1F, EE_1 appartiennent respectivement à l'équateur géographique, à l'équateur instantané et à l'écliptique, l'angle F sera γ', l'angle E, $\pi-\theta$, l'angle E_1, θ_1 ; le côté EF, ζ, angle compris entre le colure des solstices et le grand cercle des deux pôles ; le côté EE_1, $\psi_1-\psi$.

Ce triangle donne, en s'arrêtant aux termes du premier ordre

$$\theta_1-\theta=-\gamma'\cos\zeta,\ \psi_1-\psi=\frac{\gamma'}{\sin\theta}\sin\zeta.$$

Remplaçant θ et ψ par leurs valeurs précédentes et désignant par $\Delta\theta_1$ et $\Delta\psi_1$ les quantités $\theta_1-\theta_0$, $\psi_1-(\psi_0+c_1t)$, on a, en faisant $\frac{1}{1+\mu}$ égal à $1-\mu$:

$$\Delta\theta_1=-\gamma(1-\mu)\sin(\Gamma+\phi)-\gamma'\cos\zeta,$$

$$\Delta\psi_1=-\frac{\gamma}{\sin\theta}(1-\mu)\cos(\Gamma+\phi)+\frac{\gamma'}{\sin\theta}\sin\zeta.$$

Or $$\zeta=\Gamma'+\varphi-3\frac{\pi}{2}.$$

Si l'on remplace γ' par $\gamma+c\sin(\Gamma+\phi)$, on obtient, en écrivant

θ_1 au lieu de θ, puisqu'on se borne aux termes du premier ordre :

$$\Delta\theta_1 = \gamma\mu \sin(\Gamma+\phi) - c \sin(\Gamma+\phi)\cos\zeta,$$

$$\Delta\psi_1 = -\frac{\gamma}{\sin\theta_1}(1-\mu)\cos(\Gamma+\phi) + \frac{\gamma'}{\sin\theta_1}\sin\zeta.$$

4. Comme γ est égal à $0''.15$ environ, μ à $\frac{1}{305}$, on peut négliger $\gamma\mu$; on aurait donc, abstraction faite des forces perturbatrices $\Delta\theta_1 = 0$, $\Delta\psi_1 = 0$, ce qu'admettent les astronomes.

Mais il n'en est pas ainsi dans la nature, et l'on doit écrire

$$\Delta\theta_1 = -\gamma \sin(\Gamma+\phi) + \gamma' \sin(\Gamma'+\phi) ;$$

$$\Delta\psi_1 = -\frac{\gamma}{\sin\theta_1}\cos(\Gamma+\phi) + \frac{\gamma'}{\sin\theta_1}\cos(\Gamma'+\phi),$$

ou

$$\Delta\theta_1 = c \sin^2(\Gamma+\phi) ;$$

$$\sin\theta_1 \Delta\psi_1 = c \sin(\Gamma+\phi)\cos(\Gamma+\phi).$$

On aura alors

$$\Delta\delta_1 = c \sin(\Gamma+\phi)\cos(\Gamma+\phi-\alpha_1),$$

$$\Delta\alpha_1 = c \sin(\Gamma+\phi)[\cot\theta_1 \cos(\Gamma+\phi) - tg\delta_1 \sin(\Gamma+\phi-\alpha_1)].$$

L'erreur commise par les astronomes qui posent, avec Oppolzer, $\Delta\delta_1 = 0$, $\Delta\alpha_1 = 0$, peut donc s'élever, sur la déclinaison, à près de $0''.01$ ($0''.085$).

Si l'on considère cette quantité comme négligeable, il n'en peut pas être de même de l'erreur commise sur l'AR de la polaire, par exemple.

A l'époque actuelle cette erreur peut s'élever à $0^s.027$.

Pour λ *Ursæ Min.* elle serait de $0^s.07$.

5. Il me semble donc que le système de l'axe instantané doit absolument être rejeté, et que l'on doit en revenir à celui de Laplace-Bessel, en ajoutant à leurs formules incomplètes les termes de nutation à courte période (diurne ou semi-diurne).

Et cela avec d'autant plus de raison que, dans ce dernier système, on peut définir une heure, non pas à peu près, mais rigoureusement uniforme,* tandis que, dans celui de l'axe instantané,† M. Darwin a démontré que l'heure est sujette à des variations de même ordre et de même période que les variations de latitude, multipliées, de plus, par la tangente de la latitude

* *Revision des Constantes de l'Astronomie stellaire*, p. 93.

† " Je dois rendre à Oppolzer cette justice qu'il a défini l'heure dans un méridien *fixe* et dans l'équateur géographique," *Traité des Orbites*, p. 198.

du lieu, en sorte que les heures qui seraient déterminées au même instant sous les latitudes de $+60°$ et de $-60°$ différeraient entre elles de $0^s.017$; de même que celles qui seraient observées à 12^h d'intervalle en deux lieux situés sous le 60e degré de latitude.

Si l'on tient compte de la précession, la formule des variations périodiques de l'heure donnée par M. Darwin

$$\Delta\tau = -\gamma tg\Phi \sin\Gamma$$

doit s'écrire

$$\Delta\tau = -\gamma' tg\Phi \sin\Gamma' = -tg\Phi[\gamma + c\sin(\Gamma+\phi)]\sin[\Gamma - c\cos(\Gamma+\phi)],$$

ou simplement, en négligeant les termes du second ordre,

$$\cot\Phi\Delta\tau = -\gamma\sin\Gamma - c\sin\Gamma\sin(\Gamma+\phi),$$

formule dans laquelle le second terme, très faible, a une période diurne, tandis que celle du premier est de 305 jours.

6. Dans le tableau suivant nous donnerons, quant aux variations eulériennes,

U) les formules usuelles (incorrectes) ;

I) les formules correctes relatives à l'axe instantané ;

L) les formules absolument rigoureuses de Laplace-Bessel relatives aux axes principaux, l y représentant la longitude orientale du premier méridien (axe X).

Les unes et les autres se rapportent aux observations faites dans le méridien instantané ou dans le méridien fixe. Dans ce dernier cas les signes $\pm$ s'appliquent à un passage $\frac{\text{supérieur}}{\text{inférieur}}$.

A la simple inspection de ces formules on constate immédiatement que, dans ce dernier système, la nutation eulérienne disparaît entièrement dans la somme des coordonnées de deux étoiles de même D à peu près, observées à quelques minutes d'intervalle, l'une au N., l'autre au S. ; que la différence de ces coordonnées, au contraire, double ces deux nutations en éliminant les autres : avantages bien précieux que n'offrent pas les formules relatives à l'axe instantané.

Si Oppolzer avait pû prévoir les modifications que l'introduction des forces perturbatrices apportait à ses formules si simples $\Delta\theta_1 = 0$, $\Delta\psi_1 = 0$, il est plus que probable qu'il eût renoncé à substituer le système de l'axe instantané à celui des axes principaux, qu'il a conservés, du reste, dans la définition de l'heure, ce que n'ont pas fait ceux qui l'ont suivi.

Cette définition de l'heure, base fondamentale de l'astronomie, doit être telle que l'heure soit, au même instant, la même en tous lieux, et croisse uniformément avec le temps. Et une telle définition est impossible dans le système de l'axe instantané.

Elle n'existe que dans le système des axes principaux.

Nous n'avons pas à nous occuper des formules usuelles U), qui sont incorrectes.

Si nous comparons les formules correctes I) et L), il nous semble que, même en pratique, la comparaison est toute à l'avantage de ces dernières. Dans les unes comme dans les autres, γ et Γ doivent être déterminés empiriquement. Et ce sera d'autant plus long et plus difficile qu'il existe en réalité deux nutations initiales, l'eulérienne à période de 305 j., la chandlérienne à période de 431 j., comme nous allons le démontrer ci-dessous.

Mais les formules L) ont le grand avantage de permettre la détermination de ces quantités en éliminant toutes les autres variations (nutation et aberration) et c'est une raison de plus pour y revenir.

AR. U) $\Delta\alpha_I = 0.$

I) $\Delta\alpha_I = c \sin(\Gamma + l + a)\{\cot\theta \cos(\Gamma + l + a) - tg\,\delta_I \sin(\Gamma + l)\}$

L) $\Delta\alpha = \pm\gamma\{-\cot\theta\cos(\Gamma + l + a) + tg\,\delta \sin(\Gamma + l)\}.$

D. U) $\Delta\delta_I = 0.$

I) $\Delta\delta_I = c \sin(\Gamma + L + a)\cos(\Gamma + l)$

L) $\Delta\delta = \mp\gamma\cos(\Gamma + l).$

Lat^{de} U) $\Phi_I = \Phi_m + \gamma\cos(\Gamma + l).$

I) $\Phi_I = \Phi_m + \gamma\cos(\Gamma + l) - c\sin(\Gamma + l + a)\cos(\Gamma + l).$

L) $\Phi = \Phi_m.$

H^{re} U) $\tau_I = \tau_0 + nt.$

I) $\tau_I = \tau_0 + nt - tg\,\Phi_I\{\gamma\sin(\Gamma + l) + c\sin(\Gamma + l + a)\}.$

L) $\tau = \tau_0 + nt.$*

7. S'il est une question d'un intérêt capital pour l'astronomie de précision, c'est bien celle de la nutation initiale. Les travaux de Chandler lui ont fait faire un pas de géant par la découverte d'une période de 431 j.

Deux illustres géomètres ont cherché à expliquer comment la période chandlérienne de 305 j., calculée pour une Terre rigide, s'est transformée en celle de 431 j. ; l'un en considérant que ce n'est pas pour la Terre entière, mais pour son écorce, qu'on doit calculer la période ; l'autre en ayant égard au renflement équatorial occasionné par la force centrifuge.

Leurs raisonnements seraient irréprochables si la période de 305 j. était déduite des moments connus C et A d'une Terre rigide.

Mais cette période a été déduite des constantes de la précession et de la nutation déterminées empiriquement pour la Terre (ou l'écorce terrestre) dans son état actuel, c'est-à-dire ayant subi l'effet de la force centrifuge.

* *Revision des Constantes de l'Astronomie stellaire*, p. 93.

Là n'est donc pas l'explication de la période de 431 j. qui existe indubitablement, mais qui ne s'est pas substituée à celle de 305 j.

Je me propose de démontrer que pour l'écorce il existe deux termes à constantes arbitraires ; la période de l'un est de 305 j., celle de l'autre doit être déterminée empiriquement.*

8. En désignant par M_x, M_y les moments des actions de l'écorce sur le noyau dans les deux plans perpendiculaires à x et à y, par A et C, A′ et C′ les moments principaux du noyau et de l'écorce dans un plan perpendiculaire à l'équateur et dans ce dernier plan, B et B′ étant supposés égaux à A et A′ ; par P et Q les moments des forces perturbatrices perpendiculaires aux axes x et y, par p, q, n les vitesses angulaires d'un point du noyau, par $p+\delta p$, $q+\delta q$, n celles d'un point de l'écorce qu'on supposera sur le prolongement du rayon du premier, les centres de gravité du noyau et de l'écorce étant censés coïncider ; les équations d'Euler s'écriront, pour le noyau

I.
$$A\frac{dp}{dt} = -(C-A)nq+P+M_x,$$
$$A\frac{dq}{dt} = (C-A)np+Q+M_y ;$$

et pour l'écorce :

II.
$$A'\frac{d(p+\delta p)}{dt} = -(C'-A')n(q+\delta q)+P-M_x,$$
$$A'\frac{d(q+\delta q)}{dt} = (C'-A')n(p+\delta p)+Q-M_y.$$

Nous avons ainsi quatre équations pour déterminer p, q, δp, δq.

9. En négligeant, dans une première approximation, $A'\delta p$ et $A'\delta q$ vis-à-vis de Ap et de Aq, et faisant $A+A' = 2A''$, $C+C' = 2C''$, la somme des équations précédentes, prises deux à deux, donnera

III.
$$A''\frac{dp}{dt} = -(C''-A'')nq+P,$$
$$A''\frac{dq}{dt} = (C''-A'')np+Q.$$

* J'ai trouvé ces deux nutations initiales dans ma *Théorie du Mouvement de Rotation de l'Ecorce solide du Globe*, 1898. On a contesté la validité de ma démonstration. J'espère que la suivante ne donnera prise à aucune objection. J'ajouterai que j'ai trouvé en plus, pour l'écorce, un terme de nutation *générale* d'une période de 431 j. également ; il n'en sera pas question dans la présente analyse sommaire.

Ne nous occupant que des constantes arbitraires, nous aurons, en faisant

$$\frac{C''-A''}{A''}=\mu\ ;$$

IV. $$p=\gamma_1\cos(n\mu t-\sigma),\ q=\gamma_1\sin(n\mu t-\sigma).$$

La différence des équations I. et II. prises deux à deux donne, en admettant provisoirement que $\frac{C'}{A'}=\frac{C}{A}$.

V. $$\frac{d\delta p}{dt}=-\frac{C'-A'}{A'}n\delta q-M_x\left(\frac{1}{A}+\frac{1}{A'}\right),$$
$$\frac{d\delta q}{dt}=\frac{C'-A'}{A'}n\delta p-M_y\left(\frac{1}{A}+\frac{1}{A'}\right).$$

Quant aux constantes arbitraires, on tirera de ces équations, en faisant

$$\frac{C'-A'}{A'}=\mu'\ ;$$

VI. $$\delta p=\gamma_1'\cos(n\mu' t-\sigma'),\ \delta q=\gamma_1'\sin(n\mu' t-\sigma').$$

10. Les termes dépendants des constantes arbitraires dans les expressions des vitesses angulaires $p'=p+\delta p$, $q'=q+\delta q$ de l'écorce seront donc

VII. $$p'=\gamma_1\cos(n\mu t-\sigma)+\gamma_1'\cos(n\mu' t-\sigma'),$$
$$q'=\gamma_1\sin(n\mu t-\sigma)+\gamma_1'\sin(n\mu' t-\sigma').$$

$\mu=\frac{C''-A''}{A''}$ provient des équations III. qui sont identiquement les mêmes que celles du mouvement de la Terre solide.

La période du premier terme des deux équations VII. est donc de 305 j. environ.*

Mais $\mu'=\frac{C'-A'}{A'}$ dépend des moments d'inertie de l'écorce, qui nous sont absolument inconnus. L'empirisme seul peut le déterminer.

Et puisque M. Chandler a trouvé une période manifeste de 431 j., nous avons admis que $\frac{C'-A'}{A'}=\frac{1}{431}$.

Il existe donc pour l'écorce terrestre deux nutations initiales, l'une eulérienne, l'autre chandlérienne.

* Je dis environ parce que j'ai négligé $A'\delta p$ vis-à-vis de Ap.

11. J'en ai constaté l'existence dans de longues séries d'observations. L'amplitude des variations dans les latitudes de Greenwich de 1880 à 1891, réduites par MM. Thackeray et Turner, est de 1″·15.* En en faisant les sommes à 5 et à 7 mois de distance, elle a été réduite respectivement à 0″·77 et à 0″·845.

La série des latitudes de Poulkova, publiée par M. Ivanof, nous a conduit à une période de 430 j. environ pour la nutation chandlérienne, de 290 j. pour l'eulérienne.†

Or, et j'insiste sur ce fait qui démontre empiriquement l'influence, niée par les astronomes, de la nutation initiale en AR′, *j'ai déduit de la comparaison des observations de Struve en AR* (1824) avec celles de Lindhagen en AR également et de Peters en D (1843),‡

304·8 j. et 318·6 j. pour la période eulérienne,

447·2 j. et 460·3 j. pour la période chandlérienne.§

Les coefficients que j'ai trouvés par la série de M. Ivanof sont

chandlérien	eulérien	annuel
0″·13	0″·09	0″·08

D'après M. Chandler ils seraient

0″·16	0	0″·11

La valeur trop forte de ses deux coefficients provient de la négligence de la nutation eulérienne.

Il nous semble nécessaire que ces coefficients, ainsi que les deux périodes eulérienne et chandlérienne, soient déterminés à nouveau, puisqu'il n'a jamais été tenu compte que de l'une des deux seulement dans les recherches sur la variation des latitudes.

* *Bulletin de l'Académie royale de Belgique*, novembre 1898.

† *Bulletin de l'Académie royale de Belgique*, avril 1900.

‡ Vérification pratique des formules du mouvement de rotation de l'écorce terrestre (*Bulletin de l'Académie royale de Belgique*, octobre 1899).

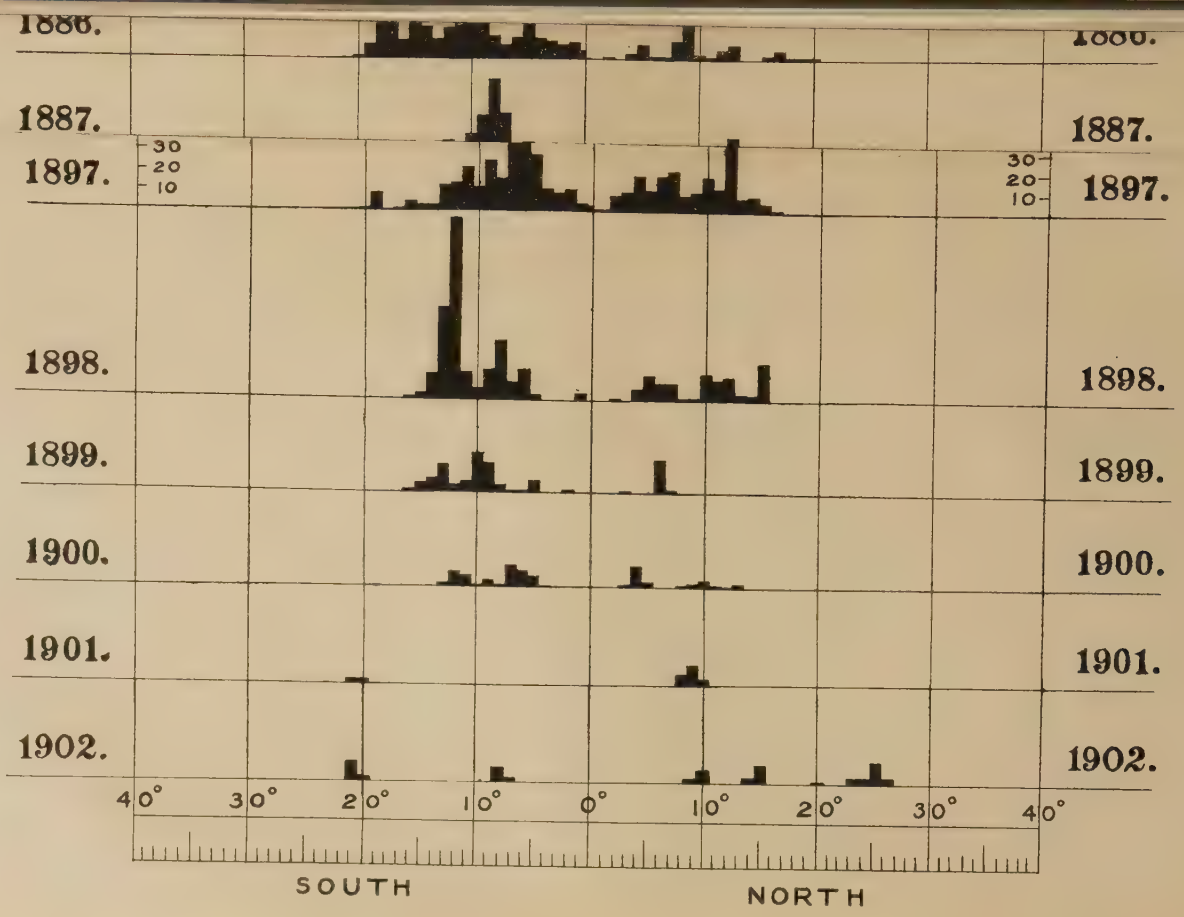
§ Les observations de Struve m'ont conduit à une période de 336 5 j. intermédiaire entre celles de 305 et de 430 j. et à une constante = 0″ 08 environ (*Revision des Constantes de l'Astronomie stellaire*, p. 13). Celles de Wagner (qui à fait usage des formules U), quoiqu'elles soient très précises, ont donné pour γ une valeur tout à fait insignifiante 0″·01, d'accord, du reste, avec les formules de réduction incorrectes dont il a fait usage (*Annuaire de l'Observatoire royal de Belgique*, 1890, p. 302).

Mean Daily Area of Sun-spots for each Degree of Solar Latitude for each Year from 1874 to 1902 as measured on Photographs at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following paper is an extension of one under a corresponding title communicated to the Royal Astronomical Society in November 1889 and printed in the *Monthly Notices*, vol. l. pp. 10, 11. The table which follows has been formed by taking out the total areas of whole spots as expressed in millionths of the Sun's visible hemisphere for each degree of solar latitude for each year and dividing these by the number of days of observation to give the mean daily area. Plate 15 presents these mean daily areas in graphical form, mean daily areas under 0·7 being omitted. In apportioning the different spots to their respective latitudes the following rule has been observed. If the heliographic latitude of the centre of any single spot, or group of spots when measured as one, showed °·5 or any higher figure in the decimal place, the entire area of the spot was taken as belonging to the next higher whole degree of latitude. If it showed °·4 or any lower figure in the decimal place the entire area of the spot was taken as belonging to the degree of latitude indicated by the integral part of the number. Thus a spot at latitude 7°·5 was taken as wholly belonging to latitude 8°, but one at 7°·4 to latitude 7°.

The diagram and the tables bring out clearly several peculiarities of the distribution of spots. First of all for the period in question, viz. the twenty-nine years from 1874 to 1902 inclusive, spots in a higher latitude than 33° were at all times rare, and when seen were never large or long-lived. Taking them as a class by themselves they were seen irregularly, appearing at times which did not seem to bear any fixed relation to any one of the four chief stages of the sun-spot cycle—minimum, increase, maximum, and decline. Omitting these spots in very high latitudes—a term which would cover a zone 10° wide in each hemisphere, from 33° to 42°, for no spots were observed in a latitude greater than 42°—the years of maximum, 1883 and 1893, showed spots in practically every latitude between 30° N. and 30° S., and they were numerous from about 8° to 24° in both hemispheres. In the years following the maximum a marked tendency was shown for the spots to appear in lower latitudes. Thus in the periods of decline, 1885–8 and 1898–9, and in the corresponding period, 1874–6, of the preceding cycle, 22° was generally the highest latitude shown. In 1876, 1888, and 1899,


1886.
1887.
1897.
1898.
1899.
1900.
1901.
1902.
30
20
10
40°
30°
20°
10°
0°
10°
20°
30°
40°
SOUTH
NORTH

Mean Daily Area of Sun-spots for each Degree of Solar Latitude for each Year from 1874 to 1902 as measured on Photographs at the Royal Observatory, Greenwich.

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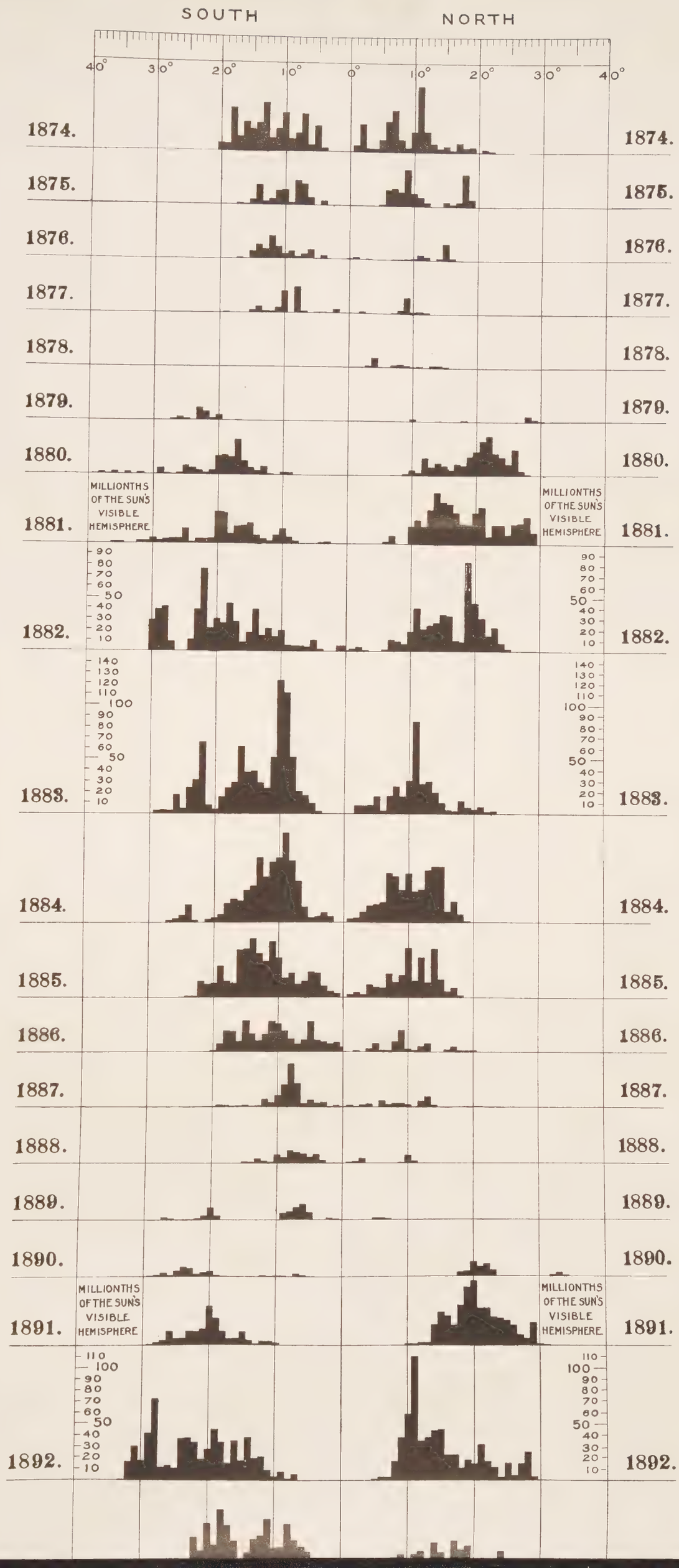
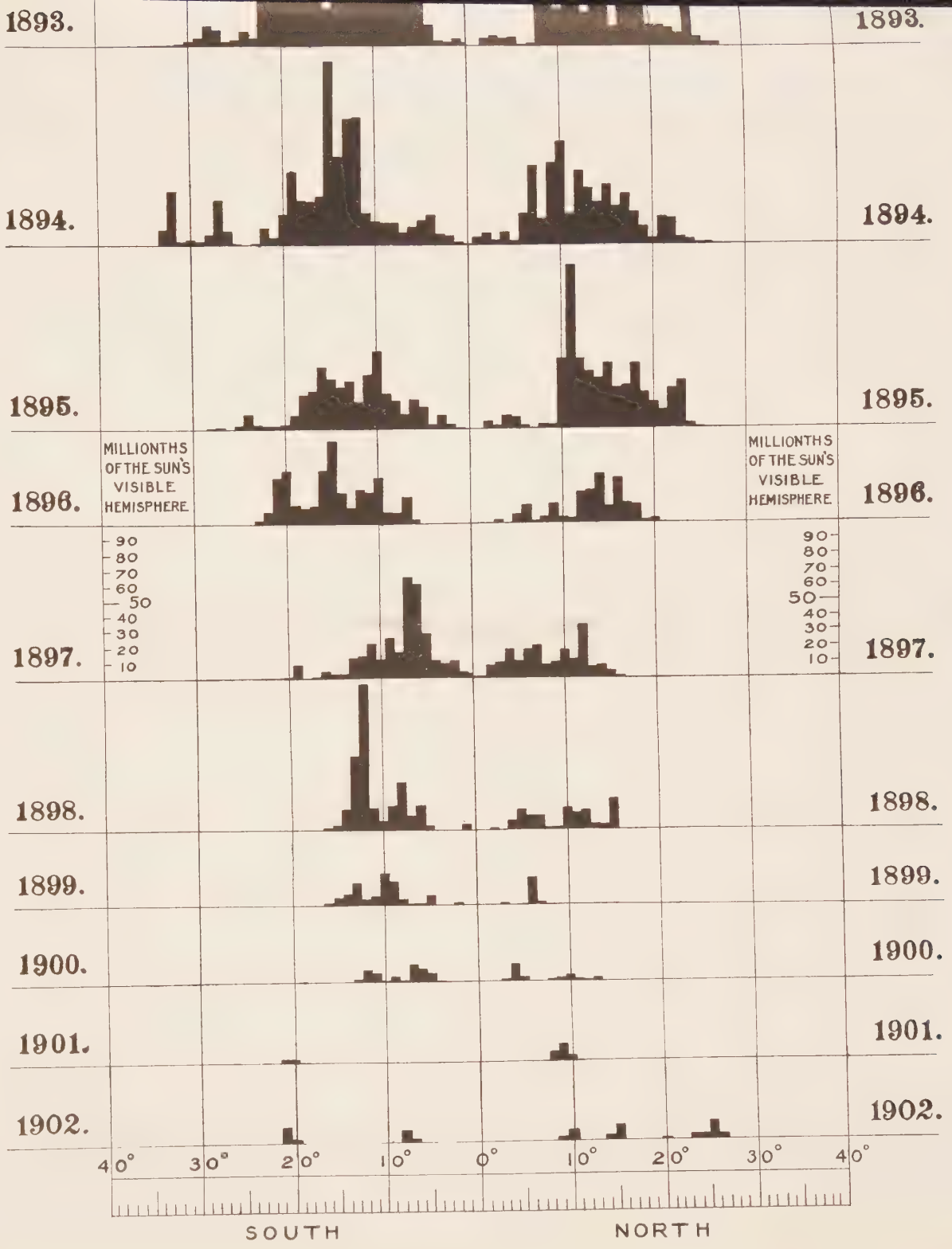


MEAN DAILY AREAS OF SUN-SPOTS FOR EACH DEGREE OF SOLAR LATITUDE FOR EACH YEAR FROM 1874 TO 1902, AS MEASURED ON PHOTO


1893.
1894.
1895.
1896.
1897.
1898.
1899.
1900.
1901.
1902.
MILLIONTHS OF THE SUN'S VISIBLE HEMISPHERE.
90
80
70
60
50
40
30
20
10
40°
30°
20°
10°
0°
SOUTH
NORTH

that is to say, about one or two years before minimum, no spots were seen outside the limit of 18° from the equator. But immediately minimum was reached the spots became more widely extended in latitude owing to the occurrence of outbreaks in high latitudes. Thus at minimum each hemisphere, considered separately, showed two clearly defined spot-zones marked off from each other by a broad belt in which there were no spots at all. This was especially marked in the years 1889 and 1890, when the very region, centering about latitude 15°, which when an entire solar cycle is considered is the most prolific of the whole solar surface, was completely free from spots.

Of these two spot-zones in each hemisphere the lower appears to correspond to the series of spots of the expiring cycle. This series, two years before minimum, was confined within the 18° limit, and would appear at minimum to seldom attain a greater latitude than 10° or 12°. The spots with a latitude of 18° to 30° or more, on the other hand, seem to be the first members of the new cycle.

During the periods of increase, as 1879–81 and 1890–2, the equatorial belt was almost wholly free from spots, indicating possibly the complete disappearance of the last members of the old cycle. At maximum, however, the spots of the new cycle were most widely spread, and were seen even in the near neighbourhood of the equator, so that at maximum, and in the first stage of the decline after it, as in 1874, 1882–6, 1893–7, the equatorial region showed its greatest activity.

The comparison of the two hemispheres shows that on the whole the southern has been the more prolific; but that the critical points of the progress of the cycle have been marked earlier by the northern spots than by the southern. Thus in the two periods of increase in 1881 and 1891 the northern hemisphere had a marked advantage over the southern, and similarly in those of decrease the drop in the spotted area in 1885 and in 1896 was much more strongly shown in the northern hemisphere.

Latitude.	1874.	1875.	1876.	1877.	1878.	1879.	1880.
+40°	...	...	...	...	...	...	...
39	...	...	...	...	...	...	...
38	...	...	...	...	...	...	...
37	...	...	...	...	...	...	...
36	...	...	...	...	...	...	...
35	...	...	...	...	...	...	0·5
34	...	...	...	...	...	...	...
33	...	...	...	...	...	...	...
32	...	...	...	...	...	0·3	...
31	...	...	...	...	...	...	0·03
30	...	...	...	0·5	...	1·0	0·1
29	...	...	...	0·2	...	2·2	0·3
28	...	...	...	...	...	5·3	1·0
27	...	...	...	...	...	0·4	4·0
26	...	...	...	...	...	0 4	23·8
25	...	...	...	...	...	0·1	10·3
24	...	...	...	...	...	...	16·7
23	...	...	...	...	...	...	20·0
22	1·2	0·2	...	...	...	...	36·4
21	2·4	...	...	...	...	...	31·0
20	0·1	...	...	...	...	0·3	21·9
19	5·0	6·4	...	...	...	...	17·1
18	4·2	29 8	...	...	...	0·9	8·7
17	8·4	4·2	...	...	...	0·1	10·2
16	1·9	1 6	2·2	...	...	0·3	4·5
15	5·8	3 8	15·3	...	0 8	...	8·6
14	3·7	...	2·0	...	2·1	0·5	11·9
13	8·0	...	0·1	...	2·1	...	7·6
12	19 7	3·4	2·8	1·5	...	...	15·9
11	62 1	8·5	4·7	2·1	1·1	...	2·3
10	19·2	12 6	1·2	1 6	0·7	2·1	4·2
9	4·2	34·7	0 9	14·8	2·0	...	1·2
8	12·1	13·4	0·4	2·7	2·6	...	...
7	39·3	16 7	0·4	1·0	2·2	...	...
6	28·6	15·2	...	...	...	...	...
5	11·6	1·6	...	....	0·6	...	...
4	3·4	0·2	...	...	9·1	0·2	...
3	2·9	0 2	0·6	0·1	1·5	...	...
2	25·8	...	0·1	1·6	...	...	...
1	5·8	0·2	1·6	...	0·1	...	...
0	...	0·1	0 1	0·2	0·1	...	...

1881.	1882.	1883.	1884.	1885.	1886.	1887.	Latitude.
							°
...	...	...	...	...	...	...	+40
...	...	...	...	...	...	...	39
...	...	...	...	...	...	...	38
...	...	...	...	...	...	...	37
...	...	...	...	...	...	...	36
...	...	...	...	...	...	...	35
...	...	...	...	...	...	...	34
0·1	...	...	...	...	...	...	33
0·1	...	...	...	...	...	...	32
0·1	...	0·2	...	...	...	...	31
0·1	...	0·03	...	...	...	...	30
11·2	0·2	0·1	...	...	...	...	29
26·9	0·1	...	...	...	...	...	28
20·8	0·04	...	...	...	...	...	27
20·0	0 4	0·01	...	...	...	...	26
8·8	2·1	0·01	...	...	...	...	25
19·0	7·0	0·3	...	...	...	...	24
19·0	20·9	2·4	0·05	...	...	...	23
6·6	12·9	2·0	0·06	0·1	...	...	22
34·2	28·4	6·3	0·1	0·2	...	0·04	21
31·4	43·2	4·1	0·3	0·4	1 6	0·1	20
19·1	82·1	6·1	1·3	0·1	1·7	0·1	19
19·5	8·5	11·8	7·4	2·1	1·3	...	18
31·0	9·7	2 6	19·0	8·8	4 8	...	17
36·9	33·3	3·4	10·1	5·4	2·0	0·2	16
41·2	33·3	9·9	51·6	16·4	0·7	0·02	15
50·1	25·9	19·7	51·5	44·3	0·7	1·4	14
34·0	23·5	29·7	48·0	13·6	7·3	9·5	13
17·2	20·0	27·5	30·9	36·4	5 0	5·6	12
23·6	38·6	84·8	28·5	15·4	1·4	1·3	11
17·0	17·7	30·4	45·1	44·9	2·7	2·2	10
0·2	6·5	15 9	30·1	20·3	20·1	3·4	9
0·4	8·9	25·2	42·4	13·9	9·4	3·7	8
7·7	10·4	16·5	45·5	21 6	1·9	2·7	7
1·9	1·7	2·8	18·3	8·6	1·7	5·8	6
...	0·1	15·8	16·4	9·6	7·6	0·4	5
...	...	7·7	15·1	11·1	2·7	3·1	4
...	1·0	7·4	9·4	2·2	0·2	2·0	3
...	3·7	7·4	3·8	4·5	1·1	0·8	2
...	2·4	0·02	3·2	2·5	0·5	1·6	1
...	0·9	0·02	0·2	0·4	0·9	0·6	0

Latitude.	1888.	1889.	1890.	1891.	1892.	1893.	1894.
+40°	...	...	..	...	...	42 0·06	...
39	...	...	...	...	...	...	...
38	...	...	...	...	...	...	...
37	...	...	...	...	...	...	...
36	...	...	...	...	...	...	...
35	...	...	...	...	...	0·1	...
34	...	...	0·9	...	...	0·02	...
33	...	...	3·6	0·1	0·1	...	...
32	...	...	1·5	0·04	0·2	...	...
31	...	...	...	0·9	0·6	...	0·2
30	...	...	...	1·5	0·3	...	0·2
29	...	...	0·2	20·5	3·5	0 02	...
28	...	...	0·1	5·5	25·1	0·3	0·1
27	...	...	0·3	8·9	14 9	3·3	0·5
26	...	...	0·4	17·8	4·3	2·9	1·1
25	...	0·1	0·7	20·0	15·4	6·2	0·9
24	...	0·3	0·5	23·2	2·9	35·1	3·5
23	...	0·1	5·7	25·8	11·7	7·8	5·0
22	...	0·02	10·4	31·4	17·0	12·6	17·4
21	...	0·01	8·1	30·2	31·3	14·2	17·7
20	...	...	12·6	46·5	14·4	14·2	3·8
19	...	...	4·4	38·4	18·1	40·7	11·7
18	...	...	3·6	32·6	11·2	35·6	21·4
17	...	...	0·4	15·9	22·5	45·3	33·1
16	...	...	0·1	27·0	22·5	13·5	25·1
15	...	...	...	29·3	44·7	34·1	38·7
14	...	...	...	17·3	43·8	44·4	27·3
13	0·2	...	...	2·2	37·8	23·4	36·4
12	0·7	0·01	...	2·4	35·1	35·5	47·4
11	2·1	0·04	...	1·2	107·4	32·6	19·6
10	7·4	0·2	...	1·1	58·1	25·3	66·3
9	0·7	...	...	0·4	37·9	32·7	52·3
8	0·2	...	...	0·02	17·3	31·6	16·8
7	0·3	0·9	0·05	...	3·0	1·6	50·4
6	0·3	1·5	...	...	3·2	0·9	19·8
5	0·7	1·5	0·02	0·04	1·2	5·4	2·5
4	0·3	...	...	0·3	0·1	5·0	8·1
3	4·2	0·05	...	0·1	...	6·5	3·1
2	1·5	...	...	...	...	5·0	7·3
1	1·0	0·1	...	0·05	...	0·4	5·0
0	0·03	0·1	...	...	...	0·5	0·5

1895.	1896.	1897.	1898.	1899.	1900.	1901.	1902.
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	0·04
...	...	...	...	...	0·1	...	0·04
...	...	...	...	...	...	...	...
...	...	...	...	...	...	0·02	...
0·02	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
0·1	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
0·03	...	...	...	...	...	...	...
0·05	...	...	...	...	...	0·01	0·04
0·1	...	0·03	...	...	...	...	0·1
0·1	...	...	...	...	...	...	3·3
0·7	...	...	...	...	...	...	11·0
3·2	0·1	0·02	...	...	...	...	2·6
30·7	0·1	...	...	...	...	...	2·8
25·7	...	...	...	...	...	...	0·4
11·9	0·1	0·04	...	...	...	...	0·3
16·6	2·2	0·04	0·1	...	...	...	0·8
23·4	1·5	0·2	...	...	...	...	0·04
41·5	11·4	0·2	0·3	...	...	...	0·2
27·9	12·8	0·03	0·5	...	...	...	0·1
26·3	28·5	1·2	...	...	...	...	...
41·8	10·5	4·8	20·4	0·2	...	...	9·7
32·3	31·1	8·1	3·4	0·2	0·03	...	2·8
36·7	20·7	7·5	3·7	...	2·0	...	...
44·7	19·8	34·4	12·9	...	0·7	...	...
105·3	3·1	12·1	11·6	0·7	1·3	0·03	...
44·8	3·4	17·9	14·4	...	3·5	3·6	6·2
25·1	12·5	10·1	1·7	0·1	2·0	10·9	1·3
3·0	4·0	8·7	0·9	0·5	0·8	6·4	...
0·8	1·6	20·9	9·5	2·1	...	0·2	...
1·9	11·7	18·2	9·4	17·9	0·1	...	...
7·2	5·9	11·5	13·4	0·1	2·7	...	...
8·6	0·1	18·7	6·3	0·4	11·0	...	...
2·3	2·2	10·3	0·1	1·0	1·4	...	...
4·6	0·5	8·2	1·2	0·2	...	0·3	...
0·2	0·04	1·2	...	...	0·5	0·4	...
0·1	0·02	1·3	...	...	...	...	...

Latitude.	1874.	1875.	1876.	1877.	1878.	1879.	1880.
− 0°	...	...	0·5	...	...	...	...
1	...	...	...	...	...	...	...
2	...	...	...	3·4	...	...	...
3	...	...	...	0·4	...	...	...
4	3·1	3·6	3·1	0·7	...	...	...
5	23·8	0·9	0·1	0·7	...	...	0·1
6	6·0	6·6	8·3	...	...	...	...
7	36·3	19·7	4·1	1·2	0·1	...	...
8	17·1	23·0	1·9	23·4	0·1	...	...
9	13·7	2·5	6·7	0·3	0·1	...	2·2
10	36·7	14·2	4·5	19·9	...	...	2·4
11	21·6	13·9	11·1	4·8	...	...	0·4
12	12·5	5·7	20·6	1·2	...	0·2	1·0
13	46·0	3·1	8·8	1·7	...	0·2	7·5
14	27·2	18·4	12·9	6·1	...	...	3·7
15	21·2	4·9	6·2	2·4	...	...	6·4
16	27·8	0·8	...	...	...	...	12·3
17	14·0	1·2	0·6	...	...	0·7	33·6
18	41·0	...	...	...	...	...	15·9
19	6·8	...	...	0·5	...	0·2	18·4
20	7·3	...	...	...	...	5·0	17·4
21	...	...	...	...	...	2·1	3·5
22	...	...	...	...	...	7·7	1·6
23	...	...	...	...	...	11·2	3·3
24	...	...	...	...	...	0·5	6·2
25	...	...	...	...	...	1·7	7·7
26	...	...	...	...	...	3·0	1·5
27	...	...	...	...	...	1·8	0·5
28	...	...	...	...	...	0·04	0·9
29	...	...	...	...	...	...	5·1
30	...	...	...	...	...	...	0·1
31	...	...	...	...	...	...	0·2
32	...	...	...	...	...	...	1·5
33	...	...	...	...	...	...	0·05
34	...	...	...	...	...	...	1·6
35	...	...	...	...	...	...	0·2
36	...	...	...	...	...	...	2·5
37	...	...	...	...	...	...	0·7
38	...	...	...	...	...	...	1·8
39	...	...	...	...	...	...	0·6
40	...	...	...	...	...	...	...

1881.	1882.	1883.	1884.	1885.	1886.	1887.	Latitude.
...	5·1	0·2	0·7	0·4	4·4	1·0	— ° 0
...	4·9	0·01	0·2	3·5	8·2	1·6	1
...	...	...	5·7	5·2	10·1	1·1	2
2·1	...	0·05	8·9	10·6	9·4	4·7	3
1·8	1·9	2·5	5·2	22·1	10·8	3·8	4
...	10·0	9·9	4·2	23·1	27·7	6·4	5
...	3·9	17·0	14·1	14·5	11·0	3·2	6
0·02	4·7	21·0	38·1	12·2	7·2	21·4	7
2·5	4·8	52·3	56·9	21·9	12·0	39·1	8
6·2	5·4	111·5	82·3	17·4	19·6	21·3	9
14·5	19·2	122·9	59·3	35·9	25·9	10·7	10
8·3	13·3	52·2	55·8	50·8	28·1	4·3	11
2·6	20·6	23·5	42·2	32·8	16·0	7·2	12
4·6	14·1	28·5	59·9	40·1	10·2	1·0	13
7 8	38·1	39·5	33·6	53·3	16·7	1·4	14
18·9	16·8	38·3	29·7	44·2	28·8	0·8	15
16·0	8·5	61·9	19·9	43·5	9·7	0·5	16
15·4	26·8	28·8	21·2	12·5	19·0	0·9	17
9·3	44·5	23·9	17·6	14·7	19·4	2·1	18
30·0	25·2	15·7	7·2	28·4	7·4	1·4	19
31·2	29·2	3·9	4·2	12·9	1·7	0·9	20
5·0	21·1	8·1	3·0	11·9	0·8	0·01	21
4·6	74·2	66·3	0·4	14·7	0·1	...	22
4·1	37·0	31·3	2·1	1·0	0·2	...	23
1·5	10·4	23·8	16·2	0·6	0·06	...	24
13·9	0·9	4·7	6·5	0·1	...	...	25
4·6	0·5	17·5	3·7	...	0·04	...	26
5 2	8·5	3·0	2·0	...	...	...	27
4·1	40·1	3·8	0·5	...	0·01	...	28
2·8	38·2	2·9	0·06	...	...	...	29
5·2	27·8	...	0·2	...	...	...	30
2·4	...	...	...	...	...	...	31
2·0	...	...	...	...	...	...	32
...	...	...	...	...	...	...	33
...	...	...	...	...	...	...	34
1·2	...	...	...	...	...	...	35
1·3	...	...	...	...	...	...	36
...	...	...	...	...	...	...	37
...	...	...	...	...	...	...	38
...	...	...	...	...	...	...	39
...	...	...	...	...	...	...	40

Latitude.	1888.	1889.	1890.	1891.	1892.	1893.	1894.
− ° 0	0·1	0·2	...	...	...	2·1	0·05
1	0·4	0·6	...	0·1	...	4·4	2·0
2	0·5	1·6	...	0·1	0·04	2·7	6·1
3	3·0	0·3	...	...	...	3·6	7·2
4	8·1	0·5	...	...	...	14·1	19·1
5	5·4	6·6	0·2	...	0·05	35·7	15·7
6	8·5	13·8	1·2	...	0·3	36·9	11·5
7	9·9	11·0	2·6	...	5·2	43·3	8·1
8	11·5	7·3	0·3	...	1·5	61·8	14·2
9	5·1	5·9	0·4	...	7·5	40·0	14·1
10	7·3	0·1	0·5	1·6	3·4	39·3	15·1
11	1·1	0·05	0·03	3·5	9·0	64·7	20·7
12	1·9	...	0·01	3·6	20·7	51·3	81·8
13	4·0	...	...	0·9	19·4	48·2	81·2
14	0·8	...	...	5·0	36·8	35·2	56·9
15	1·2	...	...	12·8	17·1	25·7	118·0
16	0·1	...	...	6·7	34·1	46·4	31·5
17	0·1	...	...	7·4	5·5	60·7	27·8
18	0·01	...	0·6	11·8	33·1	74·9	28·6
19	...	3·7	1·3	23·1	44·2	43·0	46·9
20	...	11·0	4·2	32·7	26·6	62·9	19·6
21	...	3·8	3·8	14·2	18·0	37·5	4·6
22	...	1·5	2·4	5·2	33·1	50·4	10·8
23	...	0·7	7·5	10·7	36·8	5·1	1·0
24	...	0·4	8·4	5·2	36·4	9·5	0·2
25	...	0·1	5·2	5·4	10·8	3·8	0·9
26	...	1·3	1·6	12·4	12·9	3·2	8·9
27	...	2·4	3·5	3·5	11·2	10·8	28·9
28	...	0·03	1·4	2·2	70·6	13·2	6·4
29	...	0·02	0·9	0·9	40·9	5·1	2·2
30	...	...	0·3	0·3	17·1	3·0	3·2
31	...	...	0·1	0·3	29·2	1·5	2·1
32	...	...	0·1	0·2	15·8	0·5	34·8
33	...	...	0·1	0·3	0·4	0·1	9·7
34	...	...	0·3	0·2	0·3	0·4	...
35	...	...	0·1	0·1	0·01	0·6	...
36	...	...	...	0·1	...	...	...
37	...	...	...	0·02	...	...	...
38	...	...	...	...	...	...	...
39	...	...	...	40° 0·02	...	...	...
40	...	0·03	...	42° 0·02	...	...	...

1895.	1896.	1897.	1898.	1899.	1900.	1901.	1902.
...	0·03	3·1	0·3	...	...	...	...
0·8	0·05	4·0	4·0	0·02	0·03	...	...
2·9	0·05	11·3	0·04	1·3	0·01	0·1	...
8·8	0·3	9·5	0·3	0·1	0·05	...	...
2·7	0·4	11·6	0·5	0·6	0·8	0·2	...
14·5	0·2	28·9	3·0	6·3	6·1	0·5	...
18·9	2·8	60·9	15·5	1·0	8·7	0·3	...
9·7	17·9	65·4	9·6	1·2	11·4	...	1·8
18·1	7·4	17·1	32·1	4·0	1·1	0·4	6·9
22·8	8·1	26·1	14·0	16·1	3·4	0·4	...
49·5	30·2	12·3	6·3	21·2	0·6	0·04	...
35·0	20·9	22·3	14·9	6·0	5·9	0·01	...
17·5	22·7	11·9	96·5	3·8	8·0	...	...
31·0	10·8	13·1	49·2	15·1	2·0	...	...
26·8	20·4	2·9	14·1	7·5	0·1	...	...
32·4	53·6	3·1	3·4	4·9	...	...	0·01
39·6	34·7	4·7	1·4	0·1	...	...	...
24·3	12·1	1·2	0·2	...	...	...	...
21·8	10·5	0·04	0·5	...	...	0·1	0·2
8·9	12·7	8·9	0·2	...	...	0·1	0·01
2·8	34·1	1·2	0·03	...	...	2·4	1·8
1·9	29·8	...	...	...	0·02	2·1	10·0
1·9	7·8	0·1	...	...	0·05	0·04	...
2·7	2·3	...	...	...	...	0·02	...
9·3	...	...	...	...	...	...	...
2·5	...	...	...	...	...	...	0·01
0·2	...	...	...	...	...	...	0·01
1·2	...	...	...	...	...	...	...
0·6	...	0·2	...	...	...	...	...
0·02	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
0·01	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	0·1
0·01	...	...	...	...	...	...	...
...	...	...	...	...	0·01	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	0·01	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...

Royal Observatory, Greenwich:
1903 *May* 8.

Areas of Faculæ and Sun-spots compared with Diurnal Ranges of Magnetic Declination, Horizontal Force, and Vertical Force as observed at the Royal Observatory, Greenwich, in the years 1873 *to* 1902.

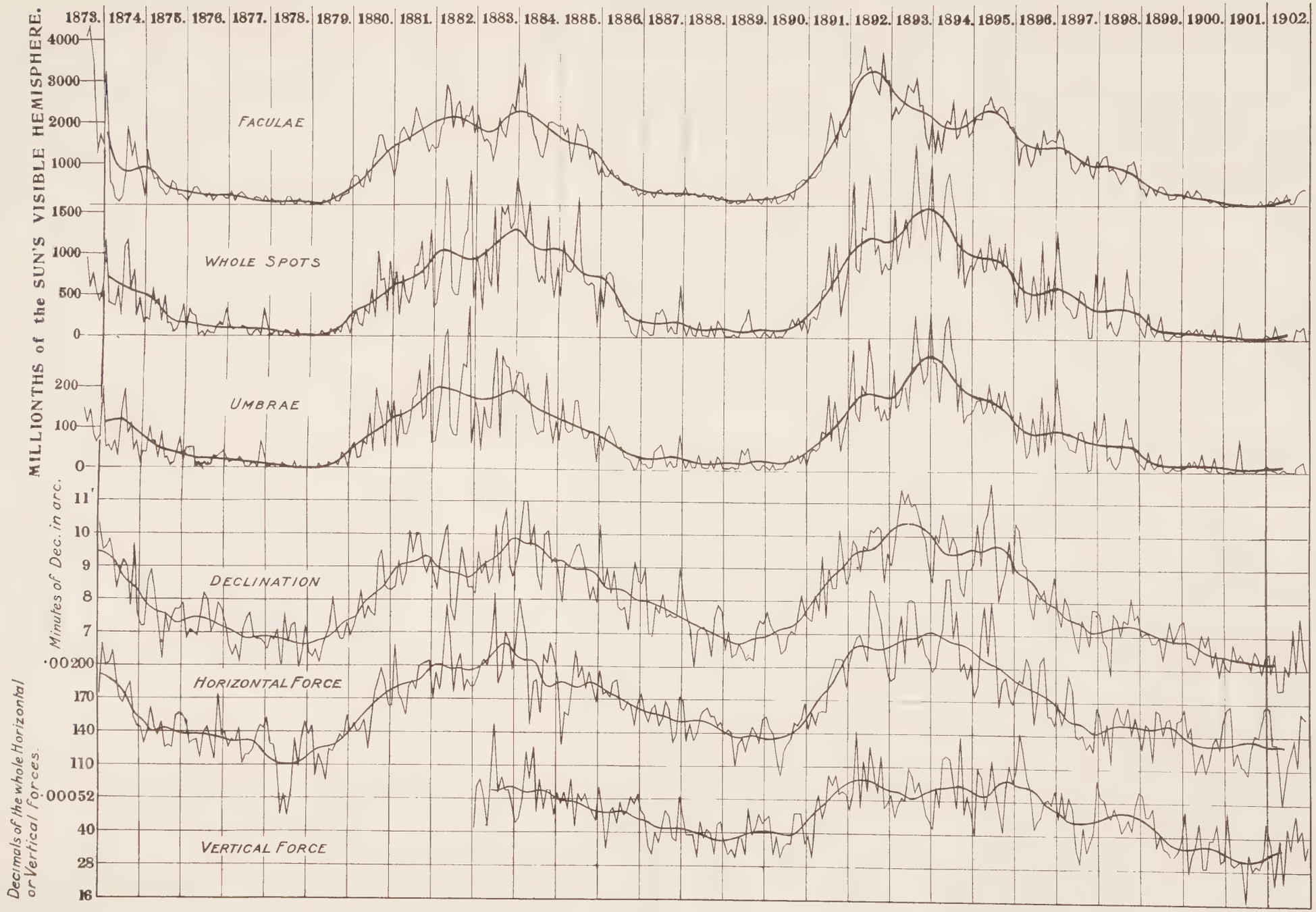
(*Communicated by the Astronomer Royal.*)

The following paper is an extension of one under a corresponding title communicated to the Royal Astronomical Society in November 1889 and printed in the *Monthly Notices*, vol. l. pp. 8–10. As in the plate accompanying that paper, which gave results up to 1888, so in Plate 16, attached to this, the three upper curves represent the areas of faculæ, whole spots, and umbræ respectively, and exhibit in a graphical form the results given on pages 106, 107, and 108 of the *Greenwich Spectroscopic and Photographic Results* for 1884, and in similar tables in the volumes for the succeeding years. For 1888 and subsequent years these results have also been communicated year by year to the Royal Astronomical Society, the figures for 1888 appearing in Table I. in the *Monthly Notices*, vol. xlix. p. 380. The mean areas for each of the three orders of solar markings have been formed by taking the means of the areas as measured upon the solar photographs, corrected for foreshortening, and reduced to millionths of the Sun's visible hemisphere for each day of observation throughout each synodic rotation of the Sun. The commencement of each rotation is defined by the coincidence of the assumed prime meridian with the central meridian, the assumed prime meridian being that meridian which passed through the ascending node at mean noon, on 1854 January 1, and the assumed period of the Sun's sidereal rotation being 25·38 days.

The ordinates for the three curves represent, therefore, the mean daily areas for each synodic rotation expressed in millionths of the Sun's visible hemisphere. The scale on which the ordinates have been drawn is five times as large for the nuclei as for the whole spots, and twice as large for the whole spots as for the faculæ, in order that the variations in each curve might be equally distinct ; for the areas of the whole spots are, on the average, about five times as great as those of the nuclei, and of the faculæ rather more than twice as large as of the whole spots. A smooth curve has also been drawn for each of the three orders of phenomena, the ordinates for which correspond, at any given rotation, very nearly to the mean of the areas for the given rotation and the six preceding and following rotations, representing thirteen rotations, or nearly a year.

From 1873 to the end of 1881 only the photographs taken at the Royal Observatory, Greenwich, were available for measurement, but from 1881 December 21 the gaps in the Greenwich

AREAS OF FACULÆ AND SUN-SPOTS, COMPARED WITH DIURNAL RANGES OF MAGNETIC ELEMENTS AS OBSERVED AT THE ROYAL OBSERVATORY, GREENWICH, 1873 TO 1902



series have been filled as far as possible by photographs taken at Dehra Dûn, India ; and from the beginning of 1885 by photographs taken at the Royal Alfred Observatory, Mauritius, as well. The daily record for the years 1882 to 1902 may, therefore, be considered as practically complete.

The magnetic curves are constructed from Table XIV. in the annual volumes of Greenwich magnetic and meteorological observations, giving the monthly mean diurnal range for declination, horizontal force, and vertical force. The values for vertical force are not available until the year 1883 in consequence of difficulty with the temperature correction. The results, both for horizontal force and for vertical force, are corrected for temperature. Now the magnetic diurnal ranges are subject to an annual period, being greatest in summer and least in winter, the period being one which, depending on geographical position, has not necessarily any counterpart in the solar spot variation, and which, therefore, should be eliminated in order to make comparison with the Sun-spot curves. The mean monthly values of diurnal range having been found from the results for the years 1874 to 1891, for declination and horizontal force, and from the results for 1883 to 1901 for vertical force, the differences in each case between the several mean monthly values and the mean yearly value give corrections which, applied to the monthly values for any individual year, clear out the average annual inequality. In this way the curves of diurnal range of magnetic declination, horizontal force, and vertical force given in the plate are found.

The smoothed curves are formed as follows. Assuming equality in the length of the several calendar months in each element, the mean of the first twelve values is taken, then the mean of the second to the thirteenth value, the mean of the third to the fourteenth, and so on. Finally the mean of each adjacent pair of the means so formed is taken, from which resulting value the smoothed curve is formed.

The declination range is given in angular measure, and those of horizontal force and vertical force in parts of the whole horizontal and vertical forces respectively, the scales being so arranged that equal changes of absolute magnetic force are represented by an equal length of ordinate. 1′ of declination corresponds to ·0003 of horizontal force, and to $\frac{\cdot 0003}{\tan \text{dip}} = \cdot 00012$ of vertical force, the values adopted in setting out the various scales.

Royal Observatory, Greenwich:
1903 *May* 8.

Mean Areas and Heliographic Latitudes of Sun-spots in the year 1902, deduced from Photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxii. p. 378, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn, India, and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1902; and Table II. gives the same particulars for the entire year 1902, and the thirteen preceding years for the sake of comparison. The areas are given in two forms: first, projected areas—that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1902 the mean daily area of the whole spots, and the mean heliographic latitude of the spotted area, for spots north and for spots south of the equator; together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results from 1889 to 1901 being added as in the case of Table II. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9.

The assumed prime meridian is that which passed through the ascending node at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotation are given in Greenwich civil time, reckoning from mean midnight.

No photographs have yet been received for four of the days included in the preparation of the following tables, but the director of the Mauritius Observatory has reported that photographs were taken there on those occasions, and that they have already been despatched to England. He further reported that the Sun was clear of spots upon three of the days. For the remaining day, values have been assumed for the areas and positions of the spots by taking the means of those obtained for the day preceding and for that following.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	1901. d							
645	Dec. 14·20	28	20	122	20	11	67	26
	1902.							
646	Jan. 10·53	26	9·2	67	73	6·1	45	92
647	Feb. 6·87	26	5·8	30	48	4·2	22	60
648	Mar. 6·21	27	37	224	195	2·9	179	210
649	Apr. 2·52	26	0·0	0·0	80	0·0	0·0	92
650	Apr. 29·77	25	2·9	21	103	3·0	21	122
651	May 27·00	27	11	44	184	6·7	28	244
652	June 23·20	27	0·1	1·0	62	0·1	0·9	62
653	July 20·40	22	0·0	0·5	23	0·0	0·3	25
654	Aug. 16·62	25	0·6	3·9	29	0·4	2·6	35
655	Sept. 12·87	27	35	198	230	16	146	279
656	Oct. 10·15	27	29	181	294	20	127	326
657	Nov. 6·45	27	35	222	320	24	166	346
658	Dec. 3·76	26	0·3	5·9	314	0·3	3·9	342

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1889	360	17·9	103	107	13·1	78·0	131
1890	361	21·3	133	273	15·5	99·4	304
1891	363	120	745	1322	86·2	569	1412
1892	362	255	1596	3230	186	1214	3270
1893	362	327	1983	2287	234	1464	2404
1894	364	317	1728	1666	231	1282	1877
1895	364	237	1330	2059	169	974	2278
1896	364	127	745	1243	90	543	1410
1897	364	122	695	977	88	514	1149
1898	363	93	532	767	64	375	891
1899	364	27	159	297	18	111	337
1900	360	22	101	150	17	75	180
1901	359	14	41	23	8·6	29	29
1902	349	14	86	150	10	62	172

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
	1901. d			°		°	°	°
645	Dec. 14·20	28	0·3	26·70	67	7·95	− 7·82	8·02
	1902.							
646	Jan. 10·53	26	0·0	...	45	7·41	− 7·41	7·41
647	Feb. 6·87	26	2·1	22·88	0·7	15·53	+21·58	22·63
648	Mar. 6·21	27	17·9	24·46	0·0	...	+24·46	24·46
649	Apr. 2·52	26	0·0	...	0·0	...	...	...
650	Apr. 29·77	25	2·1	25·95	0·0	...	+25·95	25·95
651	May 27·00	27	2·8	22·82	0·0	...	+22·82	22·82
652	June 23·20	27	0·9	26·08	0·0	...	+26·08	26·08
653	July 20·40	22	0·0	...	0·3	25·70	−25·70	25·70
654	Aug. 16·62	25	2·6	27·50	0·0	...	+27·50	27·50
655	Sept. 12·87	27	7·8	13·06	6·8	20·95	− 2·80	16·74
656	Oct. 10·15	27	4·0	9·88	8·7	20·72	−10·99	17·27
657	Nov. 6·45	27	16·5	14·81	1·2	20·59	+14·55	14·85
658	Dec. 3·76	26	3·4	21·23	0·5	18·95	+16·46	20·96

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1889	360	5·0	7·26	73·0	11·90	−10·68	11·61
1890	361	53·1	22·20	46·3	21·75	+ 1·73	21·99
1891	363	40·1	20·49	16·9	19·91	+ 8·52	20·31
1892	362	60·7	15·09	60·7	21·69	− 3·29	18·39
1893	360	51·7	14·91	94·1	14·26	− 3·93	14·49
1894	364	54·3	12·31	73·9	15·56	− 3·75	14·18
1895	364	56·5	14·26	40·9	12·54	+ 3·01	13·54
1896	364	20·3	13·60	34·0	14·77	− 4·15	14·33
1897	364	19·6	8·32	31·8	7·73	− 1·62	7·96
1898	363	11·0	9·82	26·6	10·77	− 4·75	10·49
1899	364	2·3	6·18	8·8	10·43	− 6·95	9·54
1900	360	2·6	6·61	4·9	8·34	− 3·12	7·74
1901	359	2·2	8·59	6·6	16·27	+ 2·82	10·37
1902	349	4·2	18·81	2·1	15·35	+ 7·38	17·65

The principal features of the record for 1902 are :—

1. The increase in area, which has been shown alike by umbræ, whole spots and faculæ, indicates that the minimum is definitely over and that the period of revival has begun.

2. For the whole spots the mean daily area has been more than double what it was in 1901, but has not been quite equal to that for 1900.

3. There has been an increase in the area of the umbræ, but this has been only slight.

4. The increase in the area of the faculæ has been very striking, especially during the last four rotations of the year.

5. Comparing the whole spots of the two hemispheres, the area for the northern has been just double that for the southern, but the rate of increase has been slightly the larger for the southern hemisphere.

6. The southern hemisphere was in a condition of practically undisturbed quiescence for eight rotations in succession.

7. The distribution of spots in latitude has been very characteristic of the beginning of a new cycle. The majority of the spots in both hemispheres were in high latitudes, extending from 17° to 28°, and there was one instance in each hemisphere of a group in a latitude considerably higher still. The equatorial region was entirely deserted, no group having a latitude of under 7° ; but there was a distinct belt of activity just beyond that limit, whilst there were none in the zone extending from 10° to 14°. Regarding the spots in each hemisphere as divided into two distinct regions by this barren zone, their areas and mean latitudes were :

	Spots North of the Equator.		Spots South of the Equator.	
	Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.
		°		°
Low latitude spots ...	7·4	9·85	8·7	7·74
High „ „ ...	34·1	20·74	12·2	20·81

The highest latitude recorded was 39°.

8. The days without spots were 248 in number, as compared with 289 in 1901 ; indicating an increase of activity indeed, but much less decidedly than the comparison of areas does. The earlier rotations of the year furnished much the largest portion of these spotless days, whilst rotations 655 to 659 were the only rotations of the year in which the number of days without spots did not greatly exceed one half of the total number of days of observation.

9. The number of separate groups of spots was considerably greater than in 1901, the numbers being 14 for 1901 and 35 for 1902. Of these 35, 24 groups were in the northern hemisphere and 11 in the southern.

Royal Observatory, Greenwich :
1903 *June* 11.

The Spectra of Sun-spots in the Region B–D.

By the Rev. A. L. Cortie, S.J.

The present paper is in continuation of that published in the *Memoirs* (vol. l. p. 30–56), which discussed the observations made of the spectra of ninety Sun-spots in the years 1882–89 at the Stonyhurst College Observatory. Since that period only occasional observations of the spectra of Sun-spots have been secured—some twenty-four in all—covering the period 1890–1901. The dates of the several observations were :—1890 September 16, 1891 August 18, 30, September 3, 4, 7, 23, 28 ; 1894 November 30, December 3 ; 1896 June 15, November 5, 6, 8 ; 1898 March 11, October 8, 10 ; 1899 March 22, 24 ; 1900 March 11, October 22 ; and 1901 May 22, 23, 24. The instrument employed was the Browning automatic prism spectroscope, a dispersion of twelve prisms of 60° being in most cases used. The spectroscope was attached to the 15-inch Perry memorial refractor. The method of observation was first to pass the region B–D in review, so as to pick out the lines most affected, and then to study some particular portion of the spectrum. This latter operation is exceedingly tedious and laborious, so that a detailed study of the whole of the region B–D has only been possible on one or two occasions. In the above list of dates the most complete observations were obtained on 1891 September 3, 4, 7. For the identification of the lines the beautiful photographic maps of Mr. Higgs were used. The results of the observations are collected in the following table, which gives a list of 300 lines affected in Sun-spots in this region of the spectrum. The first column gives the wave-lengths of the lines according to Rowland's values in his "Preliminary Table of Solar Spectrum Wave-lengths." Lines observed in the spectrum of the chromosphere by Professor Young are marked with an asterisk. They are taken from his revised list (Scheiner's *Astronomical Spectroscopy*, Frost's edition, p. 423). The origins of the lines are also taken from Rowland's table, as also their intensities. Intensity 1 corresponds to a line just visible on Rowland's map, the intensity of H and K on this scale being 1,000. Below 1 the successive orders of faintness are indicated by successive zeros. The third column gives the number of times each line of the list has been observed, and the fourth its mean widening, estimated as far as possible in tenths of the normal width of the line. Lines of which the widening is 1·0 and over would correspond to the most widened lines of other observers. A column is reserved for remarks on the several lines. In former papers on this subject, and especially in the *Memoir* (*loc. cit.*), the wave-lengths of the lines were taken from the reports of the British Association. These differ by about one unit from Rowland's values now adopted.

Table I.

Lines between D and B widened in the Spectra of Sun-spots.

Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
* 5890·19	D_2 Na	8	0·9	30	For discussion see notes.
91·72	A (ωυ)	1	0·5	0	
91·88	A (ωυ)			4	
93·10	Ni	3	0·3	4	Darkened once.
* 96·16	D_1 Na	8	0·9	20	For discussion see notes.
5900·14	A (ωυ)	1	0·3	2	
00·26	A (ωυ)			4	
05·90	Fe	1	0·2	4	
* 14·34	Fe	1	0·2	4	
30·41	Fe	1	0·4	6	
38·27	A (ωυ)	1	1·0	0	High Sun line. Very much darkened in spot.
46·22?	A (ωυ)	1	0·8	3	Very much darkened in spot.
52·94	Fe	2	0·8	4	
53·39	Ti			1	
56·92	Fe	2	0·5	4	
58·10	A (ωυ)	1	0·8	1	
58·46	A (ωυ)			1	
58·84	...	1	0·5	1	
66·06	Ti A?	2	0·9	2	Once much darkened in spot.
68·50	A (ωυ)	2	0·5	2	
71·56	A (ωυ)	1	1·0	1	
75·58	Fe	3	0·4	3	
77·01	Fe	3	0·4	4	
78·77	Ti	3	1·3	1	Twice the widening extended through spot into photosphere.
83·91	Fe	3	0·3	5	
85·04	Fe	3	0·3	6	
87·29	Fe	3	0·4	5	
89·51	A (ωυ)	3	0·7	0	Once unaffected in spot.
* 91·60	...	1	0·5	2	
96·96	Ni	2	0·5	1	
98·00	Fe	2	0·6	2	
99·92	Ti A (ωυ)	1	1·0	0	
6003·24	Fe	2	0·5	6	
6005·77	Fe	1	0·0	1	

	Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
	6008·19	Fe	4	0·5	4	
	08·79	Fe	3	0·6	6	
	12·45	Ni	3	1·0	1	
	13·72	Mn	5	0·6	6	
	16·86	Mn	5	0·6	6	
	18·52	...	1	1·0	0	
	20·23		4	0·6	2	Seen as one line.
*	20·40	Fe			4	
*	22·02	Mn	4	0·6	6	
*	24·28	Fe	5	0·5	7	
*	27·27	Fe	4	0·5	4	
	30·11	A (ωυ)	1	0·5	0	
	31·24	...	1	0·5	∞	
	34·27	A?	2	0·8	0	
	35·58	A?	2	0·8	0	
	36·69	A?	2	0·8	0	
	39·95	V	7	1·0	0	
*	42·32	Fe	3	0·3	3	
	53·91	Ni	7	1·0	0	Seen as one line.
	54·29	A?			∞	
	56·23	Fe	3	0·2	5	
	57·48	...	2	1·5	∞	
	59·20?	...	1	3·0		Seen once.
	63·01	...	7	1·0	0	
	64·85	Ti	3	1·5	∞	
*	65·71	Fe	3	0·5	7	
	77·12	...	4	0·9	∞	
	78·71	Fe	3	0·6	5	
	79·23	Fe			2	
	90·43	Fe	1	1·0	2	
*	6102·39	Fe	7	1·0	2	This group of close lines is very difficult to separate in spots. The greater part of the widening is due to the Ca component.
*	02·94	Ca			9	
*	03·40	Fe			4	
	03·51	...			1	
*	22·43	Ca	7	1·5	10	For discussion see notes.
	25·24	...	2	0·5	1	
	6126·44	Ti	3	3·0	1	Very black in spot once. Displaced to red once.

	Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
	6128·12	Fe	2	0·8	3	Less intense over spot once. Young attributes to Ni.
	30·34	Ni	2	0·0	1	
	31·79	...	1	0·5	0	
	32·07	...			0	
	35·58	V	2	2·0	00	Very black once in spot. Widening probably due to vanadium.
	35·99	Cr			00	
*	36·83	Fe	6	0·8	8	
	37·21	Fe			3	
	37·92	Fe	5	0·8	7	
*	41·94	Fe Ba	2	0·8	7	Not in Kayser and Runge's list of the arc lines.
	42·70	...	1	0·0	1	Obliterated over the spot.
	45·23	...	2	0·3	2	
	47·95		1	0·8	2	
*	48·04	Fe			3	
*	49·46	...	2	0·5	2	
	50·36	V	1	0·8	0	
	51·05	...	2	1·5	0000	Very faint double.
	51·55	...			0000	
	51·83	Fe	2	0·7	4	
*	54·44	Na	2	3·0	2	Displaced to violet once.
	55·35	...	1	0·0	7	Less dark over the spot.
	56·24	...	1	0·2	00	
	57·95	Fe	2	0·5	5	
	59·59	...	1	0·8	0	
*	60·96	Na	9	1·5	3	For discussion see notes.
	61·50	Ca			4	
*	62·39	Ca	3	0·6	15	Unaffected once.
	63·77	Fe	3	0·8	1	
	63·97	Ca			3	
	65·58	Fe	4	0·5	3	Unaffected once. Less dark over spot once.
	66·65	Ca	5	0·8	5	
	69·25	Ca	5	0·8	6	Darkened once.
	69·78	Ca	4	0·8	7	,, ,,
	70·42	V	4	0·9	0000	
	70·73	Fe Ni			6	,, ,,
*	6173·55	Fe	4	0·5	5	,, ,,

Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
*6175·58	Ni	4	0·6	3	Darkened once.
* 77·03	Ni	4	0·6	5	„ „
80·42	Fe	4	0·5	5	„ „
83·78?	A (ων?)	1	1·0	0	In Higgs's map solar altitude 43°
85·92	Fe	3	0·9	1	The British Association Catalogue line 6187·26 seems misplaced.
86·93	Ni			2	Unaffected once.
88·21	Fe	4	0·5	4	„ „
90·61	...	2	0·9	ooo	A faint double.
90·87				oooo	
91·39	Ni	4	0·5	6	
* 91·78	Fe			9	Unaffected once.
94·63	...	2	0·5	o	
95·63	...	2	0·4	o	„ „
99·40	V	6	3·0	o	Very dark in spots twice.
99·98	...	2	1·0	oooo	
*6200·53	Fe	3	0·5	6	
04·83	Ni	3	1·0	1	Darkened once.
10·90	...	1	1·0	oo	
12·28	...	3	2·0	oo	Faint double.
12·48	...			oooo	
13·64	Fe	3	0·9	6	Widening possibly due to the vanadium line at 6214·08.
* 15·36	Fe	2	1·0	5	
* 16·57	...	4	1·0	1	Young attributes bright line to vanadium.
* 19·49	Fe	3	0·5	6	
* 21·01	Fe	2	0·6	o	
* 21·55	Fe			oo	
24·20	Ni?	3	0·4	1	
24·72	V	1	1·0	ooo	
26·95	Fe	4	0·7	1	Obliterated over spot once.
29·44	Fe	2	0·5	1	„ „ „
* 30·94	V Fe	2	0·6	8	In the arc spectrum of iron (Kayser and Runge).
* 32·86	Fe	4	0·7	3	
33·72	...	1	1·0	oooo	
37·53	...	4	0·3	3	Unaffected once. Obliterated once.
*6238·60	...	3	0·0	2	Unaffected twice. Obliterated once.

Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
6240·17	...	5	1·0	oo	
40·53	...	4	0·9	oo	
40·86	Fe	3	0·5	3	Once widened into penumbra.
43·06	V	16	3·0	ooo	Always much widened. For discussion see notes.
43·32	...	4	0·1	1	Unaffected three times.
44·03	...	4	0·1	2	„ „
46·54	Fe	4	0·2	8	„ twice.
* 47·77	...	5	0·5	2	„ „
52·05	V	7	0·9	oo	
52·77	...	4	0·5	7	
54·05	...			co	Close triple. Unaffected once.
54·38		9	0·7	1	The metallic lines in this region of the spectrum generally much widened in spots.
54·46	Fe			5	
56·17	Fe	9	0·7	oo	Double. Observed as one line in spots.
56·57	Ni Fe			6	
58·32	Ti	8	0·9	2	
58·57	V	...	...	oooo	
58·93	Ti	7	0·8	3	
59·80	...	1	2·0	oo	
61·32	Ti	6	1·0	1	
65·35	Fe	6	0·5	5	
66·55	...	3	2·5	ooo	Faint line much widened at maximum.
67·04	...	1	1·0	oooo	Faint line much widened at minimum.
69·08	V	4	2·0	ooo	Faint vanadium line widened at maximum and minimum.
70·44	Fe	3	0·2	3	Once unaffected.
71·49	Fe	2	0·5	0	„ „
74·17	...			ooo	Fuzzy set of lines in spots.
74·87	...	6	1·5	oo	Not much darker in low than in high Sun.
75·48	A (ωv)			ooo	
78·30	O	1	0·3	4	Head of the Alpha group, principal line.
79·08	O	1	0·8	2	
79·31	O			3	
79·95	...	2	0·5	0	Unaffected once.
6280·11	O			2	

Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
6280·60	O	4	0·9	2	
80·83	Fe			3	
82·93	Co O	5	0·7	2	
86·03	A (ωυ)	3	1·0	oo	Hazy over spot once.
86·36	...			o	
87·95	O	1	2·0	1	
90·43	O	3	0·6	2	
91·18	Fe			4	
92·38?	O	4	0·4	2	Darkened once. Faint vanadium line at 6293·03.
95·39	O	3	0·3	3	Hazy over spots twice.
96·17	O	1	0·5	3	
98·01	Fe	2	0·4	5	
99·44	Fe O	4	0·3	3	„ „
*6301·72	Fe	4	0·5	7	
02·21	O	3	0·3	2	Unaffected once.
* 02·71	Fe	3	0·5	5	
02·98	O	1	0·6	2	
06·02	O	6	2·0	2	Generally much widened. Widening seemed on violet side once. Once extended into penumbra.
06·78	O	6	0·3	2	Generally not affected.
10·10	O	2	0·4	2	Darkened once.
10·85	O	2	0·0	1	„ „
11·45	...	1	0·0	oo	
11·72	Fe	2	0·3	1	Unaffected once.
12·46	...	1	1·0	oo	
12·98	...	1	1·0	ooo	
14·88	Ni	4	0·6	4	Darkened twice.
15·20	O			o	
* 18·24	Fe	4	0·6	6	„ „
18·92	...			1	
22·91	...	4	0·6	4	„ „
24·10	A (ωυ)	3	0·3	oooo	Darkened once.
24·71	O			co	
27·82	Ni	2	0·2	2	
30·32	Cr	2	1·3	1	
31·07	Fe	2	0·5	2	Unaffected once.
6332·18	...	1	0·8	o	

	Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
*	6335·55	Fe	4	0·4	6	Darkened twice.
*	37·05	Fe	4	0·4	7	„ „
	39·10	Fe	4	0·4	2	
	39·34	Ni			2	„ „
	42·60	A (ωυ)	2	1·0	oooo	Hazy over spot once.
	44·37	Fe	4	0·6	4	
*	47·31	...	3	0·4	2	Unaffected once.
	50·92	A (ωυ)	1	0·8	oooo	
	55·25	Fe	3	0·3	4	
	58·90	Fe	4	0·4	6	
	61·03	Ni	2	0·8	o	Hazy over spot once.
	61·42	...	2	0·0	oooo	Hazy over spot once. Unaffected once.
	62·56	Zn	1	2·0	1	
	63·09	Cr Fe			2	
	64·58	Fe	1	0·8	1	
	64·92	...			o	
	66·71	Ni	1	0·8	o	
*	69·68	Fe	1	0·3	o	
*	71·57	Fe	1	0·5	1	
	78·47	Ni	2	0·6	2	
	80·96	Fe	4	0·7	4	Spot band observed in this position.
	83·93	...	1	0·8	o	
	84·87	...	2	0·6	1	
	85·95	...	2	0·6	o	
	88·63	...	1	0·8	ooo	Spot band observed in this position.
	92·75	...	1	1·0	o	
*	93·82	Fe	8	0·5	7	Almost reversed once. Weakened once. Chromospheric intensity 2 [C line = 100] (Young).
*	6400·22	Fe	8	0·6	8	Almost reversed once. Weakened once. Chromospheric intensity 2 (Young).
	00·54	Fe			2	
	05·98	...	3	0·9	co	
	07·52	...	1	1·0	o	
	08·23	Fe	7	0·6	5	Darkened once.
	11·87	Fe	7	0·6	7	„ „
	15·20	...	6	0·9	1	Obliterated four times.
*	6417·13	Fe	6	0·9	1	„ „

	Wave-lengths.	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
	6420·17	Fe	6	0·5	4	
	21·57	Fe	6	0·7	7	
	21·74	Ni			000	
	31·07	Fe	6	0·5	5	Darkened once.
*	32·90	Fe?	3	0·7	1	Not in Kayser and Runge's list of arc lines.
	36·63	Fe?	1	0·8	0	
*	39·29	Ca	7	0·8	8	Darkened once.
	49·36	...	1	0·5	0	
	50·03	Ca			6	Close triplet in spots.
	50·40	Co	6	0·8	0	
	50·55	Co			0	
	55·23	Co	4	0·8	0	Darkened once.
	55·82	Ca			2	
*	56·60	...	3	0·0	3	Unaffected twice. Darkened once.
	59·12	A	1	0·5	000	
	59·90	A	1	0·8	0000	
*	62·78	Ca	7	0·8	5	Darkened once. Young doubts which component reversed. Arc line of Fe at 62·95 (Kayser and Runge).
	62·97	Fe			3	
	64·90	...	1	1·0	00	
	68·12?	A	1	0·5	000	
	69·41	...	3	0·5	2	
	71·89	Ca	4	0·6	5	
	72·70	A (ωυ)	1	1·0	00	
	73·41	A (ωυ)			00	
	75·44	A (ωυ)	3	0·6	0	
	75·85				2	
	79·41	A (ωυ)	1	1·0	00	
	80·29	A (ωυ)	1	0·8	1	
	82·10	...	1	0·6	3	
	83·03	Ni	1	0·5	1	
	83·47	A (ωυ)			1	
	87 01	A (ωυ)	1	0·5	0	
	91·88?	...	1	0·8	1	
	94·00?	Ca	5	0·8	6	
*	95·21	Fe	4	0·8	8	
*	6497·13	Fe	2	0·8	4	

	Wave-lengths	Origin.	No. of Times observed.	Mean Widening.	Intensity.	Remarks.
	6499·88	Ca	3	0·9	4	
	6514·96	A (ωυ)	1	0·5	2	
*	16·31	...	1	0·5	1	
	16·86	A (ωυ)	1	0·5	2	
	17·32	A (ωυ)	1	0·5	0	
	44·14	A (ωυ)	1	0·3	2	
	46·48	Ti Fe	3	0·4	6	
*	63·06	H	10	0·0	40	C line. Generally either reversed or less dark over spots.
	64·45	A (ωυ)	1	0·5	0	
	69·46	Fe	1	0·8	5	
	72·23	A (ωυ)	2	0·9	1	
	73·03	Ca?	1	0·8	1	
	75·27	Fe	1	0·8	2	
	81·45	...	1	0·8	0	
	86·55	Ni	1	0·5	1	
	91·58?	Fe?	1	0·5	00	
	93·16	Fe	1	0·5	6	
	94·12	Fe	1	0·5	4	
	6604·84	...	1	0·8	1	
	33·99	Fe	1	0·5	2	
	43·88	Ni	1	0·5	5	
	63·70	Fe	1	0·5	3	
	6678·24	Fe	1	0·8	5	

From these observations it appears that the chief phenomena in the spectrum of Sun-spots may be classified as follows:—Widening of lines, darkening of lines without widening, displacement of lines, obliteration of lines across spots, extension of the widened appearance right through the penumbra of spots on to the photosphere, reversal of lines, hazy fringes to lines, spot-bands, while with regard to the general absorption given by the spot there occur sometimes local darkenings, as, for instance, on 1896 November 8, when such a darkening of the general absorption was observed extending on each side of the D lines for a distance equal to half the distance between them. This want of uniformity in the general absorption, as well as the hazy fringes to some lines, especially those due to sodium, and notably the D lines, as also the spot-bands occasionally seen, were treated of in a paper printed in *Monthly Notices*, vol. xlvii. No. 1. The same phenomena have been recorded in the observations tabulated above, and in the same positions, so far at least as the fuzziness

surrounding the lines of sodium are concerned. The spot-bands, too, concur at least in the case of the region in which they were seen, and in the actual position of that near λ 6380·96, with those before recorded. They were observed on 1896 November 8. The former observations of these bands were made in 1885 and 1886, and it is possible that they are characteristic of Sun-spots at the period just after maximum.

Besides the C and D lines, often reversed, the line due to calcium at 6122·43, which is also a chromospheric line, was just reversed once in the spot of 1891 September 7. Around the line was a hazy appearance in the spot, which was also observed on September 4. The two strong iron lines at 6393·82 and 6400·54, which are also chromospheric lines, were almost reversed on 1896 November 6, and considerably weakened over the spot on November 8.

With regard to the D lines, they were also reversed on 1896 November 8 and displaced. Such displacements are not unusual in these two lines, and the characteristic hazy fringes about the widened lines are generally present.

The C line of hydrogen is very rarely widened in spots, but it is either unaffected or, as is generally the case, weakened where it crosses the spot, and frequently reversed. It also suffers twistings and displacements. A careful placing of the different portions of spots across the slit has shown that the reversals are mostly due to bridges or faculous gaps in the spots ; but on 1898 March 11 the line was beautifully reversed in the umbra and penumbra of a spot, and of a spindle-shaped appearance like a normal widening of a line. On this same date F was reversed similarly to C, but not so brilliantly.

The lines most frequently obliterated in spots are 6415·20 and 6417·13, while 6142·70 was obliterated once.

The prevalence of vanadium in the spectrum of Sun-spots has been treated of in a paper printed in *Monthly Notices*, vol. lviii. No. 7. The most remarkable line in the spectrum of Sun-spots in this region is the very faint vanadium line at 6243·06, which occurs among the most widened line in all Sun-spots at all periods of solar activity ; sometimes even with great dispersion it is impossible to see it in the photospheric spectrum, but it stands out, and generally intensely black, in the spectrum of the spots. At times in the spots it equals in intensity the strong iron line at 6246·54 ; on 1894 November 30 it was more intense than the iron line ; and on two occasions the widening extended through the penumbra.

In the table it will be noticed that among the widened lines are some attributed to atmospheric water vapour. In the first place the character of the widening of such lines is different in appearance from the widening observed on the metallic lines, which in the latter case is black, spindle-shaped, and sharp cut, while in the case of the water-vapour lines it is of a fuzzy appearance. Secondly, these lines occur generally in crowded

parts of the spectrum, so that the widening may be due, not to the water-vapour lines, but to faint solar lines very close to them in position. At the same time the possibility of the presence of water vapour in the upper regions of the Sun over Sun-spots is not excluded.

The same remarks may perhaps apply to the oxygen lines observed widened in the *a* band. Commenting upon similar observations in the *a* band made at Greenwich, Scheiner (*Astronomical Spectroscopy*, Frost's edition, page 179) remarks that "it must not be forgotten that an apparent broadening of hazy bands like *a* will always occur when the background of continuous spectrum becomes dark." But some of the lines in the band, notably 6306·02, are sharply widened and darkened. Moreover, Runge and Paschen have identified one triplet in the red end of the solar spectrum as due to oxygen.

The relative importance of the various elements, represented by their lines in this portion of the spectrum, in the spectra of Sun-spots is shown in the following table.

The first two columns give the total number of lines and total number of observations of such lines for each element; the third column gives the mean widening; the fourth the mean intensity of the lines, individual intensities below 0 on Rowland's scale being reckoned as negative; and the remaining columns the number of bright lines seen in the chromosphere and the atomic weight of each element.

TABLE II.

Relative Widening of Lines of each Element.

Element.	Total Number of		Mean		Bright Lines.	Atomic Weight.
	Lines.	Observations.	Widening.	Intensity.	Chromosphere.	
Vanadium	11	58	3·52	− 0·5	1?	51·3
Chromium	3	5	1·72	+ 0·7	0	52·2
Sodium	4	27	1·26	13·8	4	23·0
Titanium	9	30	1·15	1·6	0	50·0
Calcium	12	66	0·99	6·8	5	40·0
Cobalt	4	21	0·78	+ 0·5	...	58·7
Nickel	22	73	0·66	2·3	2	58·7
Oxygen?	20	57	0·66	2·0	...	16·0
Manganese ...	3	14	0·60	6·0	1	55·0
Iron	97	310	0·58	4·2	33	56·0
Unknown	24	66	0·48	2·1	8	...
Unknown and faint	37	79	1·06	− 1·0		...

The preponderating importance of vanadium in the spectrum of Sun-spots is evident from this table, and yet its lines are very faint in the normal solar spectrum and but one is doubtfully bright in the spectrum of the chromosphere. Passing over

chromium, of which element only three lines occur in this part of the spectrum, and sodium, which has four lines, the next important element is titanium, which has no corresponding bright lines in the chromosphere. It may be safely asserted that faint lines of vanadium and titanium occur among the most widened lines in the spectrum of Sun-spots in the region under discussion at all periods of solar activity. The calcium lines, though strong lines and coincident in five cases out of twelve with bright chromospheric lines, are generally well widened. There is not much variation in the widening at different epochs, though it is slightly more marked at the minimum.

Of the ninety-seven lines attributed by Rowland to iron in this part of the spectrum thirty-three, or about one-third, occur in Young's list of bright chromospheric lines. Of these thirty-three lines, again, twenty-six are found in Kayser and Runge's list of lines in the arc spectrum of the metal. Hence we may infer that the lines widened in spots which are also bright in the chromosphere are mainly arc lines. The question at once arises as to whether there is any difference in the behaviour of these special arc-spot-chromosphere lines in Sun-spots from the other lines which are also affected in spots. A list of these special lines was made and their mean intensity in the Fraunhofer spectrum, and their mean widening in spots was obtained. The numbers are 4·5 and 0·59, compared to 4·2 and 0·58 when all the lines of iron are averaged. Therefore there would appear to be no difference in the behaviour of such lines treated as a whole from the other iron lines. The iron lines, and in general the more prominent metallic lines, were more pronounced in the spectra of spots observed in the minimum years 1898, 1899, and 1900; but there was no displacement by them of the faint lines of unknown origin, nor more particularly of the vanadium and titanium lines in these spots. In this part of the spectrum at least there is no evidence of "crossing points" between iron lines and faint lines, and no evidence at all of change in the materials that constitute a Sun-spot. The spectrum of a Sun-spot is a very complex phenomenon in which the change in the relative widenings of lines is of minor importance when compared to the constancy of the more characteristic appearances.

Stonyhurst College Observatory:
1903 *June* 8.

On a probable relationship between the Solar Prominences and Corona. By William J. S. Lockyer, M.A. (Camb.), Ph.D. (Gött.), F.R.A.S., Chief Assistant, Solar Physics Observatory.

Introduction

In a recent communication * to the Royal Society, Sir Norman Lockyer and I gave the results which had been deduced from a minute investigation of the percentage frequency of prominences as determined from observations made by Secchi and Tacchini at Rome, and Ricco and Mascari at Catania and Palermo.

It was there shown that the chief centres of prominence action indicated movements in heliographic latitudes, the general tendency of these being in a direction towards the solar poles, and not towards the equator as is the case with the spots. Attention was also drawn to the fact that these centres of prominence activity were not restricted to narrow zones like the spots, which only occur between the latitudes $\pm 5°$ and $\pm 35°$, but that at times they were numerous in such high latitudes as $\pm 80°$ and even higher.

The object of the present communication is to give an account of the results of a general survey regarding the connection between the changes of position of these centres of prominence action and the various forms of the corona as observed during total eclipses.

It has been suggested, and the idea is generally accepted, that the various forms of the solar corona are intimately connected with the variation in the spotted area of the Sun's surface. Thus, generally speaking, at about the epochs of Sun-spot maxima, the corona is apparently very irregular in shape, there being little or none of the exquisite tracery at the Sun's poles which is so evident at the epochs of Sun-spot minima, while the streamers are less confined to mid-solar latitudes and the region nearer the equator than they are at the minima.

The facts that Sun-spots do not appear nearer the poles than latitudes $\pm 35°$, and that large coronal streamers and prominent rays are sometimes situated in much higher latitudes than these, in fact at times very near the poles of the Sun, and consequently outside the regions of spot activity, suggested that the occurrence of prominences, very important factors in the mechanism of the solar atmosphere, might be closely connected with them.

Classification of Coronas.

For the present general inquiry the forms of the coronas that have been observed since the year 1857 have been divided into three main types, and this classification, which is not new,†

* *Roy. Soc. Proc.*, vol. 71, p. 446.

† *Solar Physics*, Lockyer (Macmillan & Co., 1874), p. 278 *et seq.*; also

is one into which most of the coronas, with the exception of two, namely 1865 and 1885, can be easily placed.

Since the forms of coronas are dependent chiefly on the positions of the coronal streamers, the three different types here adopted refer in the main to the positions of these streamers in relation to the solar equator and poles.

In the first or "polar" group, as it has here been called, since streamers are found near the solar poles, all those coronas are included which seem to have no regular form. The typical features of this group are that the polar rifts are absent, being replaced to a great extent by streamers at, or very close to, the poles, and the streamers are numerous in nearly all solar latitudes; also there is no definite equatorial extension.

To this class the following coronas belong: 1860, 1870, 1871, 1882, 1883, and 1893.

In the third or "equatorial" group, since the streamers are chiefly equatorial, the form of the corona is very regular. The polar rifts have a large spread in latitude and are well defined, while the large streamers are restricted to the regions near the equator; in fact, the great equatorial extensions are best seen in this type. This form generally takes the shape of a "wind vane," and is often referred to as such. The coronas which come into this category are those of 1867, 1868, 1878, 1889 Jan., 1889 Dec., 1900, and 1901.

The second group of this classification may be termed the "intermediate" type, as the streamers are about half-way or intermediate between the poles and the equator. In this group the polar rifts are present, but they are not so extensive in latitude as in the "equatorial" class. The coronal streamers also approach nearer the polar regions than in the "equatorial" class, but not so close as in the "polar" group, while the equatorial extensions are not in such great evidence. Generally speaking this form of corona is due to a large streamer in each quadrant, which gives the corona the appearance of a square, hence the name "square corona."*

The coronas which fall under this heading are 1858, 1869, 1874, 1875, 1886, 1887, 1896, and 1898. It may be stated that the "polar" and "equatorial" coronas are always followed by an "intermediate" type, the order being polar, intermediate, equatorial, intermediate, polar, &c.†

The "intermediate" type may sometimes approach in form a "polar" or an "equatorial" type, according as the epoch of the

Bull. de l'Académie Impériale des Sciences de St. Pétersbourg, 5e. Série, vol. vi. 1897, Hansky.

* *Solar Physics*, p. 276.

† It may be here remarked that the "intermediate" type between an "equatorial" and "polar" type has only once (1869) been recorded during the period here under investigation, and this is due to the absence of eclipses during the two short available periods since that date—namely, 1879–1881 and 1890–1892.

occurrence of the eclipse occurs nearer or farther from the epochs of occurrence of polar prominences.

Further, the "intermediate" type preceding a "polar" type will differ to some extent from one immediately following a "polar" type, because the latitudes of the centres of prominence action in each case are different, as can be seen from the accompanying plate.

Two coronas which have not yet been classified are namely those of 1865 and 1885. The former of these is of a type between the "intermediate" and "equatorial," while the latter falls between the "polar" and "intermediate" groups.

The Method of Inquiry.

The first natural and crucial test to apply, in order to determine whether there was a connection between prominences and the different forms of the corona, was to inquire at what epochs the coronal streamers were situated nearest the solar poles, and whether these were coincident with those times when the prominences were most numerous in those regions.

If this relationship were found to hold good, the next step was to see if it were possible to connect the two other main types of coronas, with the other conspicuous prominence changes.

The comparison for the first test showed that the only five "polar" coronas recorded since the year 1869, when prominence observations were commenced, occurred at those epochs when the prominences attained their highest latitudes.

This satisfactory result indicated a very probable cause and effect between prominences and the coronal streamers, for the region considered was quite outside the zone of the spots, and therefore independent of them.

It was next found that the other two types of coronas were closely associated with the number and latitudes of the centres of prominence action. Thus the "equatorial" type only occurred when there was *one* definite centre of prominence action in each hemisphere, while the "intermediate" type has been recorded at those times when *two* centres of action in each hemisphere were in progress.

The accompanying illustration (Plate 17) shows the relationship between the Sun-spot curve for both hemispheres together, the latitudes of the centres of action of the solar prominences for each hemisphere, explained in detail in a previous communication,* and the times of occurrence of all the eclipses that have occurred since the year 1857. When two eclipses of the same type occur in the same, or two successive years, they have been inserted either one above the other or obliquely to avoid overcrowding. A curve is also drawn through the different types, showing their relation to the Sun-spot curve.

* *Roy. Soc. Proc.*, vol. 71, p. 447.

Since the systematic prominence observations only commenced in the year 1872, the dotted portions of the curves previous to that date are intended only to give a rough idea of the variations as based on a general repetition of the observations of 1872 to 1885.

Fortunately for the present inquiry Respighi* made some very valuable prominence observations during the years 1870, 1871, and 1872, which are sufficiently numerous to indicate the positions of the centres of prominence activity for these years. These showed that during the years 1870 and 1871 there were two well-marked prominence zones in each hemisphere, and that the latitude of one of the zones was very high. The positions of these zones are indicated in the accompanying plate by the small circles against these years, and they agree well with the dotted curves representing the probable conditions as might have been expected from subsequent observations.

The different types of corona are plotted in three different horizons in the order "polar," "intermediate," and "equatorial," and the symbols adopted for each—namely, small circles with eight rays for the first, four rays for the second, and two rays for the third—are inserted at the epochs of their occurrence, according to the general time-scale for all the curves.

The continuous and broken vertical lines denote the epochs of the Sun-spot maxima and minima, as determined from a discussion of spots recorded on both hemispheres of the Sun.

The Results of the Comparison.

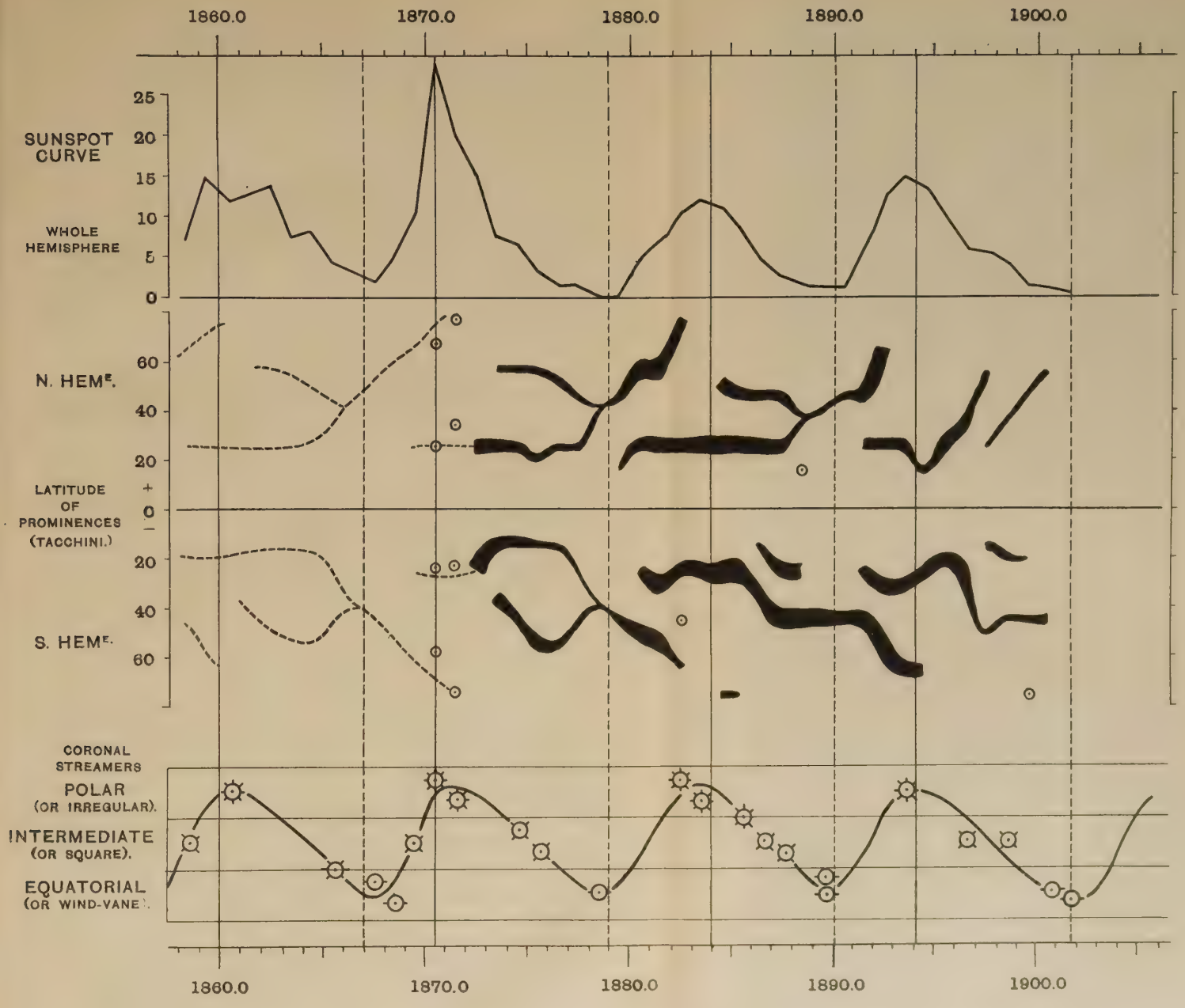
At the first glance it will be observed that the three types of the corona, as seen from the curve drawn through them, follow the Sun-spot curve very closely—that is, that at about the times of the maxima of Sun-spots, the "polar" type is present; at the minima, the "equatorial" type; and at the intervals between these, the "intermediate" type.

Although the Sun-spot curve thus affords a means of predicting in a general manner the epochs about which any of these types will occur, such a small restricted zone which the spots occupy excludes the idea of their presence being responsible for such widely distributed coronal phenomena.

The prominence curve, on the other hand, not only provides us with a more accurate method of forecast, but such phenomena can account for the changes of position and form of the coronal streamers.

By examining the prominence curves in relation to the three different types of coronas from the year 1869, this connection is seen to be very close. Thus, during the years 1870 and 1871, there were two centres of prominence action in each hemisphere, one of which was in high latitudes, and the coronas for that period

* *Solar Physics*, Appendix II. p. 654.



The vertical continuous & broken lines represent the epochs of maxima & minima of sunspot activity from a reduction of the observations of both hemispheres.

were of the "polar" type. From the year 1872 to 1877 there were two centres of prominence activity in each hemisphere, both in comparatively low latitudes, and the two eclipses during the period, namely 1874 and 1875, were of the "intermediate" type. The next eclipse, 1878, occurred when only one centre of action was in existence, and the form of the corona was of the "equatorial" type.

As these centres of prominence action reached their extreme polar limits (about $\pm$ 80°) and a new centre had in the meanwhile commenced in lower latitudes ($\pm$ 25°) the eclipses of 1882 and 1883 were of the "polar" type.

The next two eclipses, of 1886 and 1887, which were "intermediate," occurred when there were again two centres of prominence action in each hemisphere, but none near the poles. When the centres became single, as they did in the years 1889, 1890, and 1891, the two coronas observed in the year 1889 were of the "equatorial" type. With the movement of these centres to high latitudes in the years 1892, 1893, 1894, the eclipse of 1893 was of the "polar" type.

The two eclipses of 1896 and 1898, which were "intermediate" in type, occurred when there were two chief centres of prominence action, while the two most recent eclipses of 1900 and 1901 were good examples of the "equatorial" type, and were concurrent with only one centre of prominence activity in each hemisphere.

If the eclipses observed between 1856 and 1870 be compared with the dotted prominence curves for the same period, it will be seen that a similar connection seems to exist between the latitudes of the centres of action of the prominences and the three types of coronas.

The Duration of Prominence Action and the Resulting Coronal Streamers.

It may here be remarked that if the prominences, which are acknowledged to be uprushes of incandescent matter into the upper regions of the solar atmosphere, are the cause of the coronal streamers (and the evidence seems to support this idea), and consequently responsible for the existence of the coronal streamers, and therefore of the shape of the corona, then the latter should alter their forms and positions not quickly but gradually, following the slow changes of the latitude and intensity of the prominence centres of action. It is the *sum total of prominence action* in the different zones which produces the large coronal streamers, and not any particular prominence at any particular moment; it is for this reason that the form of the corona is not a fleeting phenomenon changing every minute or hour, but lasting over several months, and sometimes as much as a year or more. That the general form of the corona does undergo comparatively slow changes, is borne out to a great extent by the similarity of coronas which are observed at eclipses which occur

close together, such as those in 1900, 1901, the two eclipses in 1889, &c.

It may be suggested that the continued action of a centre of prominence disturbance in one latitude, extending over the greater part of a year or longer, would undoubtedly result in the formation of a large coronal disturbance, and form the locus of origin of a streamer situated in that region. It is also possible that this long-continued action might also disturb the solar atmosphere to such an extent that the coronal streamer might exist for a short time, even after the prominence action in this region had ceased.

The prominence curves here illustrated are determined from values which represent the mean prominence condition throughout a year. At any one moment therefore, say, during an eclipse, the prominences visible on the limb need not necessarily be situated in those particular latitudes as shown on the curves, but the probability would be that on the average they would be found near there. The non-coincidence of the position angle of a long coronal streamer with that of a large prominence is not necessarily a criterion that prominences as a whole do not cause the former.

The Structure and Formation of Streamers.

It is of great interest to briefly note the connection between the centres of prominence action when either two or one of them exist in each hemisphere. In the first place, a well-defined large coronal streamer apparently originates, as many photographs indicate, not from disturbances at the *centre* of its base, but at the two ends. Such a streamer is generally made up of groups of incurving structure, termed by Ranyard* "synclinal" groups, and this structure is in many cases very distinct. When there are *two* centres of prominence action in one hemisphere, the coronal disturbances resulting from each trend towards each other and constitute a large streamer with an apparent "arch" formation. If the two centres of prominence action exist in comparatively mid-latitudes, one large streamer is formed in each quadrant, and the form of the corona is of the "intermediate" or "square" type.

When one of the centres is near the region of the poles, and the other in comparatively low latitudes, the tendency is still for the two disturbed coronal regions to trend towards each other, but they constitute either a large streamer of an "arch" formation nearer the solar poles with a very extended base, or two separate streamers.

With *one* centre of action of prominences in each hemisphere, the resulting coronal disturbances in both hemispheres curve towards the solar equator and form apparently a large equatorial streamer: the "equatorial" type of corona is here formed.

* *Mem. R.A. Soc.*, vol. xli. p. 486.

The accompanying sketches illustrate in diagrammatic form the general relationships between the latitudes of the spot zones, the latitudes of the centres of action of the prominences, and the suggested resulting positions and origin of structure of the coronal streamers for each of the three types of coronas here discussed. It will be noticed that in the case of the "polar" and "intermediate" types, when the Sun-spots are numerous, the zones in which they occur have apparently little connection with the coronal streamers. When the latitudes of the spot zones do approximate more nearly to the bases of the coronal streamers, as in the "equatorial" type, and might be considered as being the origin of their existence, the spots at these epochs are near a minimum—that is, are very few and small in size and have the least power of action.

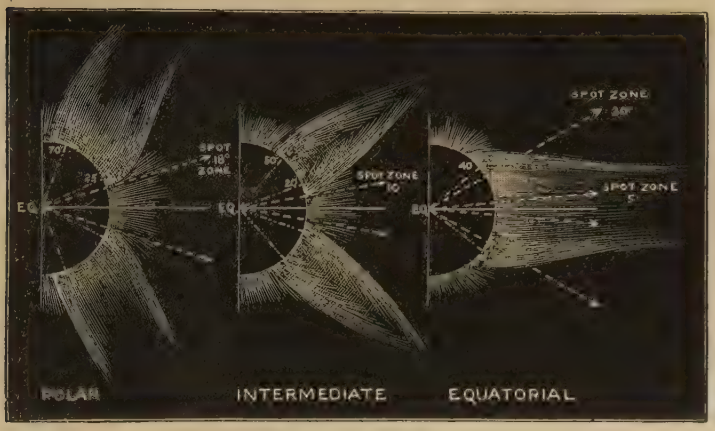


Diagram showing by short radial lines the positions of the centres of prominence action in relation to the chief features of individual coronal streamers and the general forms of the corona. The positions of the Sun-spot zones are indicated by the longer radial lines. The three types of coronas are here illustrated.

In conclusion, I wish to express my thanks to Mr. W. H. Wesley, of the Royal Astronomical Society, for his kindness in answering numerous questions relating to different coronas with which he was familiar.

Conclusion.

The following is a summary of the conclusions arrived at in the preceding paper :—

1. The "forms" of coronas may be grouped generally into three classes, here named "polar," "intermediate," and "equatorial," according as the streamers appear near the solar poles, in mid-latitudes, or about each side of the equator.

2. The sequence of these forms, if sufficient numbers of eclipses occurred, should be equatorial, intermediate, polar, intermediate, equatorial, &c.

3. The various forms of the corona are closely connected with the positions (as regards latitude) of the centres of action of the solar prominences.

4. The coronas of the "polar" or irregular type occur about the times when the prominences are most abundant near the solar poles.

5. The "equatorial" coronas occur when there is *one* centre of prominence action (about latitude $\pm$ 45) in each hemisphere.

6. The "intermediate" type is produced by two centres of prominence action in each hemisphere, but neither centres near the poles.

7. The peculiar "arched" form of some streamers is produced by the action of two zones of prominences situated near the extremities of their base.

8. Sun-spot activity has apparently no *direct* connection with the production of the coronal streamers.

Experiments as to the Actuality of the "Canals" observed on Mars.

By J. E. Evans and E. Walter Maunder.

The experiments described in the following paper were undertaken in order to ascertain whether the impression of a network of fine lines, such as forms what is now known as the "canal system" of *Mars*, could be produced upon entirely unbiassed observers without those lines having a real objective existence; and, should this prove to be the case, to find out the conditions most favourable for the creation of such an impression.

The experiments were made in the following manner. A circular disc, varying according to circumstances from 3·1 to 6·3 inches in diameter, was given to a class of boys to sketch. The boys in the class were usually twenty in number, and were seated at various measured distances from the disc. These distances varied in the extreme from 15 feet up to 62 feet, but more generally from about 17 feet to 38 feet. The boys were all supplied with a piece of drawing-paper upon which a circle 3 inches in diameter had been described, and were instructed to fill in that circle with all the details which they could perceive upon the disc. No hint was given them that they ought to see lines or dots or any other form of marking; they were simply urged to draw all that they could see and be sure of, each for himself, without noting what his neighbours were drawing. They were carefully watched during the experiment, and there can be no doubt at all that the drawings were independent; indeed, the internal evidence of the drawings themselves is sufficient to prove

the point. The room used had a glass roof, affording excellent light; and care was taken to hang the diagram in a position free from glare or shade.

The first series of experiments was made on 1902 July 1, the last on 1903 May 22, the great majority having been made in the spring of the present year.

The boys employed in the experiments were from the Royal Hospital School, Greenwich. Their ages ranged from twelve to fourteen for the most part; a few were either a little older or a little younger than these ages. All of them were wholly and entirely ignorant of the appearance of *Mars* in the telescope, and of the discussions which have taken place as to the markings on his surface. They were simply shown, what was to them, an odd-looking figure, and were told to reproduce it as well as they could.

Several suggestions have been made at different times to account for the curious network of straight lines first seen upon *Mars* in any considerable development by Professor G. V. Schiaparelli during the opposition of 1877. These may be summarized as follows:—

(1) The "canals" are not really lines at all, but are the boundaries of districts differing in colour or brightness from the surrounding surface. This suggestion was due to the late Mr. N. E. Green (*Journal of the British Astronomical Association*, vol. i. pp. 110–113; *see* also *Observatory*, vol. v. pp. 135–137 and p. 143).

(2) They are "only the summation of a complexity of detail, far too minute to be ever separately discerned." This suggestion was made by Mr. Maunder (*Knowledge* for 1894 November, pp. 249–252, and for 1895 March, p. 58); also in somewhat different terms a few years later by Signor V. Cerulli (*Nuove Osservazioni di Marte: Saggio di una interpretazione ottica delle sensazioni areoscopiche*).

(3) The canals are purely subjective, and are suggested by the outlines of the "continents" or bright regions of the planet upon which they are seen. This is in effect the inference from Mr. B. W. Lane's experiments described in *Knowledge* for 1902 November, pp. 250, 251.

There will be no need to give in detail all the experiments made, as the results in each particular class were of the same character whatever the part of the planet represented. The region especially chosen was that called in Mr. Green's chart of 1877 (*Memoirs, R.A.S.*, vol. xliv.), *Beer Continent*, which shows two very characteristic markings, viz. the *Kaiser Sea*, or *Syrtis Major*, and *Dawes' Forked Bay*. The latter was of considerable practical importance in the experiments, as the clearness and fidelity with which it was rendered formed a very useful index of the observer's powers of sight and delineation.

Experiment 1.—A diagram, 5·3 inches in diameter, founded upon a drawing made by M. Antoniadi on 1901 February and published in *Knowledge* for 1902 April. The bulk of the disc was left smooth white; the *Syrtis Major* and *Sinus Sabæus* were

in dead black, *Deucalionis Regio* in grey. Six small dots were put in in dead black roughly in the places of the following "lakes" or "oases":—

Coloe Palus	*Arethusa Lacus*
Copais Lacus	*Siloe Fons*
Ismenius Lacus	*Niliacus Lacus*

The diagram was of the most definite and hard character possible, and there is nothing in it which in the least suggests a "canal" when it is examined close at hand.

The twenty boys were stationed as follows:—

Row.	No. of boys.	Distance. Feet.	Diameter of disc. ′
a	2	25	61
b	6	29	52
c	6	35½	43
d	6	41	37

The column "Diameter of disc" gives the angle in minutes of arc subtended by the diagram as viewed from the different benches.

The boys in rows *a*, *b*, and *d* made no indications of "canals"; but of those in row *c*, five showed canals corresponding in position with the following members of the Martian "canal" system:—

Argæus	...	...	...	5	observers
Arnon	...	...	...	5	,,
Deuteronilus	...	...	...	2	,,
Kison	...	...	...	4	,,
Pierius	...	...	...	1	,,
Protonilus	...	...	...	3	,,
Pyramus	...	...	...	5	,,

To put the matter in other words, these five boys joined up some of the black dots by straight fine lines, prolonging, in some cases, these lines till they reached the limb or the black oval drawn round the north polar cap.

With regard to the perceptive ability of the boys, all, except one at the smallest distance, represented *Dawes' Forked Bay* as a double inlet; the angular distance between the points of the fork as viewed from the several benches, being

$$a\ 230''\ ;\ b\ 200''\ ;\ c\ 160''\ ;\ d\ 140''$$

One boy showed all the six dots, the smallest having to him

a diameter of 36″. For the other boys the smallest dots recorded were of the following diameters :—

34″	2 observers.	50″	2 observers.
42	3 „	54	2 „
46	3 „	60	3 „
48	2 „	70	2 „

The largest dots missed were

25″	1 observer.	42″	6 observers.
29	2 observers.	46	2 „
34	2 „	48	4 „
36	2 „		

The foregoing table is of some importance, for it shows that an isolated dot, even when dead black on a white ground, would not be perceived if of smaller diameter than about 34″, and would be generally missed if under 40″, whilst such a dot could hardly escape notice if above 50″. This conclusion was so precisely that reached in numerous later experiments that it would be a mere repetition to give the actual numbers obtained in them as well as the foregoing.

Experiment 2.—A hard black-and-white diagram like diagram No. 1, the outlines being copied from a drawing made by Professor Schiaparelli 1890 May 16 (*La Planète Mars*, p. 474). Twenty boys arranged in the same manner and at the same distances as in Experiment 1 took part. The boys were an entirely different set, and the diagram was only 3·1 inches in diameter, its angular diameter being—

$$a\ 35\tfrac{1}{2}'\ ;\ b\ 31'\ ;\ c\ 25'\ ;\ d\ 22'$$

Thus though the distances were the same as in the former experiment the angular diameter of the markings was much diminished. The distance between the points of the fork of *Dawes' Bay* was—

$$a\ 82''\ ;\ b\ 71''\ ;\ c\ 58''\ ;\ d\ 50''$$

This distance was much too small to allow of separating definition, and the *Bay* was therefore always seen as a single marking, or not shown at all. Only one dot was inserted. This was placed roughly in the position of *Arethusa Lacus*. Its diameter was—

$$a\ 41''\ ;\ b\ 36''\ ;\ c\ 29''\ ;\ d\ 25''$$

None of the boys represented it. No "canals," properly so

called, were drawn ; but of the six boys in set *d*, at the greatest distance, one drew a fine line from the *Nilosyrtis* to *Margaritifer Sinus* ; but this was a segment of a circle and concentric with the edge of the disc, whilst three others drew a straight broad streak corresponding roughly to the northern portion of the *Hiddekel*.

Experiment 3.—As before, a hard black-and-white diagram was prepared, founded in this case upon a drawing made by Professor Schiaparelli 1890 June 9 (*La Planète Mars*, p. 475). A large number of black dots of very different diameters were inserted irregularly in the diagram. The diameter of the disc was 5·95 inches. Twenty boys drew the diagram, and were stationed thus :—

Row.	No. of boys.	Distance. Feet.	Diameter of disc. ′
a	4	38½	44
b	4	43½	39
c	4	49½	34
d	4	55½	31
e	4	61½	28

Two boys, both in row *a*, drew "canals" to the number of seven or eight each. These consisted in every case of fine lines, straight or nearly straight, joining up two or more black dots, and prolonged to the nearest "sea."

Experiment 4.—The same twenty boys as in Experiment 3 were set to draw a black-and-white diagram based on a drawing made by Professor Schiaparelli 1888 May 8-10 (*La Planète Mars*, p. 423). In this diagram a large part of the disc was lightly stippled in with minute dots, far too small to be perceived even at the closest position adopted, but leaving *Elysium* bare, so that that region appeared a little brighter than its surroundings, as it actually does on the planet *Mars* itself.

The distances and the diameter of the disc were the same as in Experiment 3, but the individual boys were made to change their seats.

Five boys showed canals : two in row *a* and three in row *c*. Of the latter, two were the boys who had seen "canals" in row *a* in Experiment 3. Later experiments showed that boys who once began to see "canals" at a favourable distance were more ready to see them afterwards at unfavourable distances. These two boys inclosed *Elysium* in a rectangle of "canals" besides inserting "canals" in many other parts of the disc.

This experiment concluded those made with diagrams of dead black-and-white. Those that follow were all made with pencil-drawings on buff drawing-paper, so that the markings on them were not quite so far removed from the subdued character shown by those on the actual planet itself.

Experiment 5.—Drawing 6·1 inches in diameter based upon one by Professor Schiaparelli made 1888 June 13, and appearing in Tav. V. fig. xiv. of this sixth memoir on the planet. *Elysium* occupied the centre of the disc. Eighteen boys stationed as under took part in the observation :—

Row.	No. of boys.	Distance. Feet.	Diameter of disc. ′
a	3	17	103
b	4	$21\frac{1}{2}$	81
c	4	26	67
d	4	$30\frac{1}{2}$	57
e	3	35	50

Nine dots were inserted in the diagram, and two boys, one in row *b*, the other in row *c*, represented four of these dots by a slightly broken streak.

Experiment 6.—Drawing 6·05 inches in diameter based upon the diagram on p. 102 of Lowell's *Mars*. In this experiment the oases shown by Mr. Lowell were reproduced approximately as given by him, and three short canals were faintly indicated. Twenty boys observed, and every one without exception showed canals in addition to those really on the disc. These canals were made by joining up the oases by straight lines or by segments of circles, although in a number of cases the oases themselves were not represented. The canals were generally prolonged till they met the limb, or one of the great *Maria*. The boys were seated at the same distances as in Experiment 5, four boys in each row.

Experiment 7.—Drawing 5·8 inches in diameter based upon a drawing made by Professor Schiaparelli on 1888 May 27, and appearing in Tav. IV. fig. vii. of his sixth memoir on the planet, and showing *Dawes' Forked Bay*, the *Margaritifer Sinus*, and the *Mare Acidalium*. Nineteen boys at the same distances as in Experiments 5 and 6. No canals were drawn by them. This experiment, together with Experiments 1 and 2, was prepared with special reference to Mr. Lane's suggestion.

Experiments 8 and 9.—Drawing 6·25 inches based upon one by Professor Schiaparelli made 1890 May 16 (*La Planète Mars*, p. 474). In this experiment none of the canals shown by Professor Schiaparelli were inserted, but a number of small irregular markings were inserted at haphazard. River-like marks were drawn flowing into *Dawes' Forked Bay* and the smaller marking of the same character which Schiaparelli has represented some 30° from it at the mouth of the *Phison*. The region of *Meroe Island* was put in in half-tone.

The same diagram was used in both experiments, nineteen boys taking part in Experiment 8 and twenty in Experiment 9. Combining the two sets in one, the stations were as follows :—

Row.	No. of boys.	Distance. Feet.	Diameter of disc. ′
a	2	17	105
b	3	19	94
c	4	22½	80
d	3	24	75
e	8	28½	63
f	4	32½	55
g	4	34½	52
h	11	37½	48

The results from these experiments were very full and instructive. In row *a* the boys appeared to be just about the distance when the minute markings on the original were beginning to take the canal-like form. In row *b* one boy saw the markings in their true aspect, to another they appeared as canals, and the third saw them imperfectly as canals. Rows *c* and *d* in every instance showed some canals, some boys seeing several. In row *e* the canals are not quite so well represented, though every boy showed something of them. Row *f* showed very few canals, row *g* a fair number, whilst the great majority in row *h* showed no canals or anything like them.

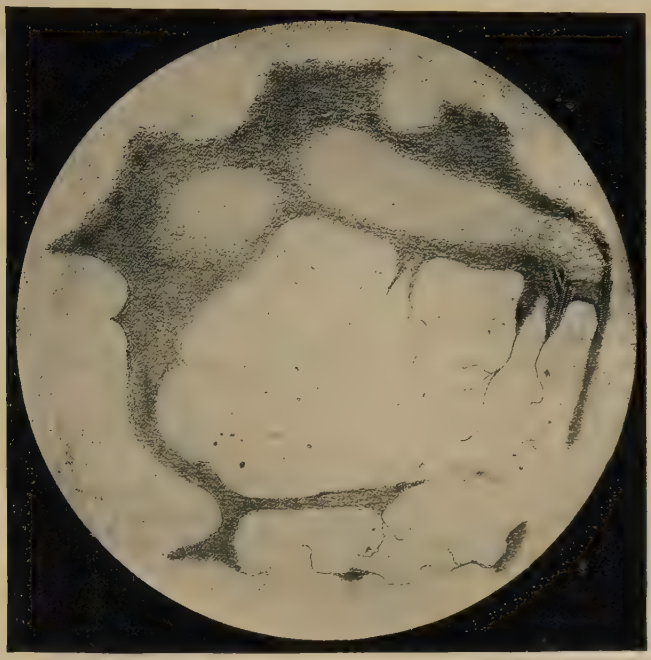
The drawings of the boys in rows *a* and *b* were especially instructive as showing that the actual details, the winding, river-like marks and the miscellaneous dots were to them just visible as such, or were just beginning to be fused into canal-like streaks.

In all twelve canals shown in the recognised charts of *Mars* were more or less faithfully indicated. Some of these were only shown by a single observer, but the six following were frequently represented:—

	a ft.	*b* ft.	*c* ft.	*d* ft.	*e* ft.	*f* ft.	*g* ft.	*h* ft.
Distance	17	19	22½	24	28½	32½	34½	37½
No. of observers	2	3	4	3	8	4	4	11
Phison	2	2	4	2	5	0	2	2
Euphrates	2	2	4	3	6	0	4	3
Hiddekel	2	2	2	2		0	2	2
Gehon	2	1	0	0	1	0	1	2
Arnon	1	1	4	1	3	2	2	4
Deuteronilus	1	1	2	0	1	0	2	0
"Allen's Canal"	1	0	2	0	0	0	0	0

Three boys, one in row *a* and two in row *c*, agreed very closely in inserting a short canal which does not appear to be in Schiaparelli's maps, and which we have here called from the name of its discoverer, "Allen's Canal." A very striking feature is shown in the drawing of this boy Allen in that he prolongs *Aeria* into

Photograph of the Original Drawing submitted to the Boys in Experiments 8, 9, 10, and 11. Diameter of the original, 6·25 inches.



KEY MAP.

Fig. 1, 17 feet.

Fig. 2, 22½ feet.

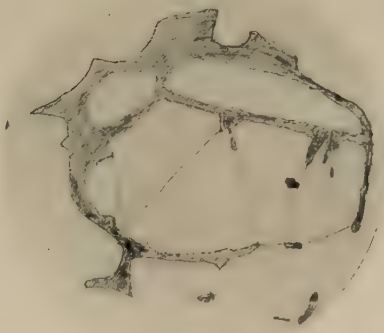


Fig. 3, 25½ feet.

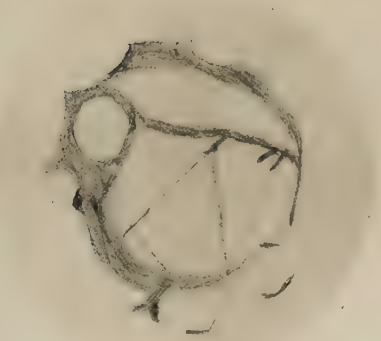


Fig. 4, 28½ feet.

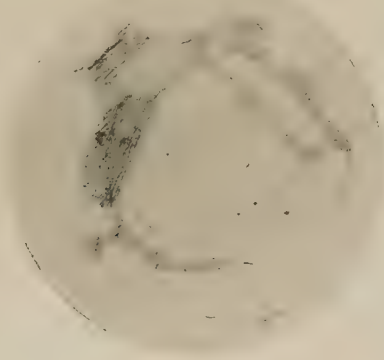
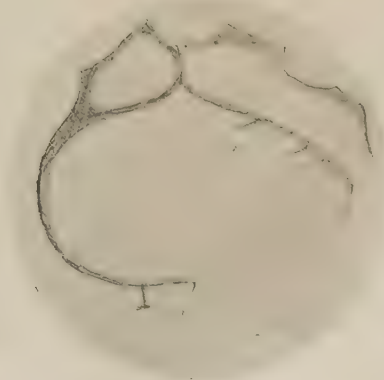


Fig. 5, 34½ feet.

Fig. 6, 37½ feet.



Photographs of the Actual Drawings made by Boys placed at the indicated distances from the original drawing.

the horn-like promontory *Solis Pons*, and cuts it off at its base by the little canal *Porus*, precisely as shown in M. Antoniadi's chart of *Mars* (*Knowledge*, 1902 November, p. 252). There was absolutely no justification in the diagram which he was copying for this marking, but clearly the presence of the two half-tone districts *Iapygia* and *Deucalion* tended to produce the impression that the narrow dark marking between them was partially split by a bright horn. Several other boys showed *Aeria* drawn out into a pointed horn, but not to the same extent that Allen did.

An important feature of this series of drawings was that eleven boys instead of drawing *Meroe Island* as a half-tone district drew the canal *Astusapes*. Only three showed *Meroe Island* distinctly as a half-tone district, and one of the three added a straight line, the canal *Astusapes* as its boundary. The entire marking, however, was not sufficiently distinctive to be clearly recognised by the more distant boys. Only one boy in row *g*, one in *h*, and none in *f* recognised the canal *Astusapes*; and none on these three rows showed any indication of *Meroe Island* as a shaded region. Of the nearer boys, therefore, 50 per cent. showed the canal *Astusapes*, and only 15 per cent. the shaded region.

The following table may prove of interest as indicating the degree of sharpness of sight possessed by the boys in their power to resolve *Dawes' Forked Bay* and the smaller fork at the mouth of the *Phison*.

Row.	No. of boys.	Dawes' Forked Bay. Angles.	Resolved by	Fork of Phison. Angles.	Resolved by
		″		″	
a	2	323	1	202	1
b	3	289	3	181	3
c	4	244	3	153	2
d	3	229	3	144	1
e	8	193	7	121	2
f	4	169	3	106	1
g	4	159	3	100	0
h	11	147	11	92	0

The table shows that the resolution was almost always effected down to 150″, the few failures being probably due merely to carelessness or inattention. Below 140″ the falling off is very marked, and none resolve the fork of the *Phison* under 105″. This result is in entire agreement with Experiments 1 and 2, and shows that whilst a single dot is generally seen down to 40″, and even a little below, the resolution of two neighbouring spurs requires a separation about three times as great.

Experiments 10 and 11 were made upon the same drawing, some of the boys in Experiment 8 being moved to new places in Experiment 10. Similarly some of the boys in Experiment 9 were moved to new places in Experiment 11.

In the two experiments taken together, twenty-three boys observed. Of these sixteen were brought forward, seven were moved further back. The results for the boys who were brought forward were of the same general character as in Experiments 8 and 9. Two boys were stationed at only 15 feet from the drawing to be copied. These and those at 17 feet were too near to confuse any details. The boys placed at middle distance perceived canals a little more readily than had been done by their comrades in the same position.

Combining these sixteen boys with the thirty-nine boys who had observed the same diagram before, and grouping together observers at distances which differed only by a few inches, the number of "canals" represented was as follows :—

Distance. Feet.	No. of boys.	Canals drawn.	Canals per observer.
15	2	4	2·00
17	8	26	3·25
19	5	16	3·20
23	9	36	4·00
$25\frac{1}{2}$	3	15	5·00
$28\frac{1}{2}$	8	21	2·63
$33\frac{1}{2}$	9	15	1·67
$37\frac{1}{2}$	11	14	1·27

The seven boys who were moved back to row *h* from rows *b* and *c*, either from memory or from the practice they had already obtained, were much more successful in seeing canals than the boys who had formerly been in this position. It would appear to be not wholly a matter of memory, for one of the seven drew an additional canal beside those which he had seen at the smaller distance. As a rule nothing like "doubling" of the canals was perceived by any of the observers. There were, however, two exceptions, and both related to the same canal, *Hiddekel*. This was clearly doubled in Experiment 9 by a boy at $37\frac{1}{2}$ feet, and in Experiment 10 by another at $25\frac{1}{2}$ feet.

Experiment 12 was upon the same original drawing as that used in Experiment 6; but whereas on that occasion Mr. Lowell's drawing had been reproduced with the oases, but with no canals, now the canals were shown but not the oases, the object being to see whether the boys would insert dots at the juncture of the canals to resemble the oases of Mr. Lowell's original drawing, and also to see whether they would detect actual canals when presented to them. With one single exception all the boys (eleven in number) showed the canals very strongly and unhesitatingly. They had not the slightest difficulty in seeing them. They were seated at $22\frac{1}{2}$ feet (four boys), $28\frac{1}{2}$ feet (four boys), and $37\frac{1}{2}$ feet (three boys). The only boy who did not represent the canals was one at the middle distance, who

showed instead a general shading over the whole district covered by the canals. As a rule oases were not shown, the exceptions being one boy at 28½ feet and two at 37½ feet. The first-named of these three was tried a second time on the back row, and showed more canals than he had done in the nearer position, beside introducing three or four more oases.

Experiment 13.—The same drawing was used as in Experiment 12, with the exception that the canals were not shown but short wavy lines were drawn, radiating irregularly from the position occupied by Mr. Lowell's oases. Ten boys observed, stationed as follows :—

Row.	No. of boys.	Distance in feet.	Diam. of disc.
a	1	15	115
b	1	19	90
c	1	24	71
d	2	28½	60
e	2	32½	53
f	3	37½	46

The boy in row *a* showed a number of canals, but was near enough to the original to see that the markings were not straight. The boy in row *b* put in two canals and some blurred markings. The boy in row *c* developed a fine canal system, showing some eighteen canals in all. The two boys in row *d* showed seven and eight canals respectively. The two boys in row *e* showed a great number of canals, one drawing being specially interesting, as it was a very fairly accurate representation of Mr. Lowell's entire system both of canals and of oases. The boys in row *f* showed little beyond a long elliptical canal, only corresponding roughly with Lowell's system.

Thus in Experiments 6, 12, and 13 the same drawing of the planet was submitted to the boys, but with the following differences :—

Experiment 6.—Oases inserted, but no canals.

Experiment 12.—Canals inserted, but no oases.

Experiment 13.—Neither canals nor oases inserted, but instead short wavy lines.

In each case many canals were drawn by the boys and some oases.

It appears to us in reviewing the entire series of the experiments that it is impossible to escape the conclusion that markings having all the characteristics of the canals of *Mars* can be seen by perfectly unbiassed and keen-sighted observers upon objects where no marking of such a character actually exists. They are in a sense truly "seen," not imagined, because they are the natural rendering by the eye of real markings of a different character.

There are several ways in which this impression of canal-like markings is caused. First, Mr. Green's suggestion that the canals are boundaries of regions of differing shade or tone receives some support. Such boundaries are capable of giving rise to the impression of canals.

Next, so far as the experiments go, the most fruitful source of the canal-like impression is the tendency to join together minute dot-like markings. If these are fairly near to each other it is not necessary, in order to produce the canal effect, that they should be individually large enough to be seen.

But it is not necessary that the markings should be dot-like, that is to say, approximately circular. They may be of any conceivable forms provided only that they are outside the limit of distinct vision and are sufficiently sparsely scattered. In this case the eye inevitably sums up the details which it cannot resolve into fine lines essentially "canal-like" in character. A slight aggregation may give the impression of a spot, or, to use the nomenclature now adopted for the planet *Mars*, an "oasis"; or, if the aggregation be greater still and more extended, to the idea of a shaded area.

So far as these experiments go they do not confirm, as such, the experiments made by Mr. B. W. Lane, or the conclusions which he reached. It was not found that the outlines of the continents of *Mars* were sufficient by themselves to give rise to the impression of the canal system. It is of course a question whether a little training and practice might not have led the boys to recognise canals upon continents which were entirely bare of all internal markings; but we were especially careful not to give the boys the slightest hint that we expected them to see straight lines or dots or anything of the kind. They were told repeatedly to draw all that they could see, but nothing of which they were not certain. And, as a matter of fact, in all these experiments whenever a boy drew a canal there was something on the original which had given rise to that impression, however unlike a canal that something might be.

It will be seen from the figures given above in the body of the paper that the distances at which the boys were stationed ranged in many of the experiments from one whereat the minutest markings were within the range of distinct vision to one so great that to most of the observers these minute markings produced no effect at all; generally speaking, the canals were best seen a little outside the limit of distinct vision.

The result of repeating the experiment with the same boys showed that practice tended to increase the readiness to perceive canals, so that in a second sketch a boy though removed to a greater distance still saw as many canals as at first, in some cases even more.

Generally speaking, the best draughtsmen, that is to say, those who most truthfully represented the salient features of the drawing, also showed the greatest numbers of canals. It is also

worth note that on the whole the agreement as to the canals was greater than the agreement as to the broad features of the original drawing.

We consider Experiment 12 an exceedingly important one, particularly when taken in connection with certain experiments upon the limits of vision not here described. Objects near the limit of vision necessarily divide themselves into two great classes—dots and lines. Now the limit of vision for a straight line is such that if it be of reasonable length its breadth may be $\frac{1}{15}$ of the diameter of the smallest perceptible dot, and it will still be seen. It follows, therefore, that if the surface of *Mars* were really covered with a series of straight lines, like those represented in the charts which we owe to Professor Schiaparelli and Mr. Lowell, there could be no doubt, no controversy, about their existence. Every observer would see them. A straight line can be seen when its angular breadth is very far indeed below the limit of vision for any other form. Consequently when a drawing of *Mars* upon which actual canals were drawn was submitted to the boys they were practically unanimous in showing them with unhesitating distinctness and certainty.

Our conclusion from the entire experiment is that the canals of *Mars* may in some cases be, as Mr. Green suggested, the boundaries of tones or shadings, but that in the majority of cases they are simply the integration by the eye of minute details too small to be separately and distinctly defined. It would not therefore be in the least correct to say that the numerous observers who have drawn canals on *Mars* during the last twenty-five years have drawn what they did not see. On the contrary they have drawn, and drawn truthfully, that which they saw ; yet, for all that, the canals which they have drawn have no more objective existence than those which our Greenwich boys imagined they saw on the drawings submitted to them.

It seems a thousand pities that all those magnificent theories of human habitation, canal construction, planetary crystallisation, and the like are based upon lines which our experiments compel us to declare non-existent ; but with the planet *Mars* still left, and the imagination unimpaired, there remains hope that a new theory no less attractive may yet be developed, and on a basis more solid than " mere seeming."

Recent Observations of Mars and Jupiter. By W. F. Denning.

On 1903 May 19 and 21 telescopic definition was very good, and a considerable amount of detail visible on *Mars*, especially on the latter night. The *Syrtis Major* appeared abnormally dark and sharply outlined on both dates, while on May 21 a very

bright region crossed its southern extremity (separating it from the seas yet further south), and similarly bright areas lay over *Libya* to the west and north of *Sinus Sabæus* on the east. The best view ever obtained here of *Mars* was on the latter occasion when the configuration* was somewhat as follows.

May 21, 9^h. Long. 281°·8.

On May 23 the planet was observed again at $8^h\ 30^m$, $9^h\ 15^m$, and $10^h\ 30^m$, when, however, definition was not very satisfactory. The *Syrtis Major* exhibited a faintness and indefiniteness of outline which was immediately evident, and which quite astonished me after the experience of previous evenings. This marking, usually the most conspicuous object in *Mars*, had become extremely feeble, as if covered with highly reflective vapours which, though not sufficiently dense to wholly obliterate the object, had suffused it with a much lighter tone. In the early part of the evening the veiled aspect it presented might be accounted for, it being some distance from the centre and near the terminator; but at $10^h\ 30^m$ it had become nearly central, and was most difficult to trace even then. Bad definition could not have induced this effect, for a number of surrounding features,

* The extreme delicacy of some of the features cannot be properly represented in a drawing of this nature, nor is it possible to include all the detail glimpsed at the best moments.

normally less conspicuous than the *Syrtis Major*, were observed; and among these may be mentioned *Syrtis Minor*, *Amenthes*, *Boreosyrtis*, *Nilosyrtis*, *Coloe Palus*, &c. On the following night, May 24, between 9^h and 10^h 30^m the *Syrtis Major* remained feeble and indeterminate, though perhaps not in the same degree as on May 23. Definition, however, was unsteady, but certain other markings were fairly well seen during short intervals, when the seeing was good.

On subsequent nights the *Syrtis Major* came on too late to be suitably observed and a prevalence of easterly winds continued to impair definition. Several times, however—notably on May 25 and 27—a strikingly luminous zone was remarked north of the *Cimmerium Mare*. In fact, during the whole of the last half of May the brilliancy of an extensive belt ranging along the northern borders of *Cimmerium Mare*, *Syrtis Minor*, and *Sinus Sabæus*, was very strongly pronounced.

It seems possible that the bright material which on May 21 appeared to have collected over the equator and on the south side of the *Syrtis Major* may have drifted rapidly northwards and occasioned the faintness of that marking on May 23 and 24.

As affording a possible corroboration of the phenomenon alluded to, the *New York Herald* of May 31 contained a telegram from the Flagstaff Observatory mentioning that a projection had been seen on *Mars* on May 26 in latitude 22° north and longitude 37° west. "Professor Lowell says it must have been an enormous cloud travelling northwards and dissipating as it went. This deduction is proved by its non-appearance next day."

I had previously noticed on May 4 a probable instance of change in another region of *Mars*. The canal *Nilus*, running east from *Lunæ Lacus*, exhibited a very well-marked division. There was a bright, glowing spot attached to the east side of the very dark *Mare Acidalium*, and a stream of luminous material apparently flowed from this centre far to S.E., crossing *Nilus* and cutting through the dusky material forming the canal. On the following night the same aspect was repeated, but less decidedly. The division in *Nilus* had not been seen by me on March 31 or April 2, when the conditions were very good and the diameter of *Mars* (14″·6) greater than on May 4 (12″·7). Nor was this feature noticed in 1886 March and April, when I made several drawings of the same region.

A bright spot in *Elysium*, immediately on E. side of *Trivium Charontis*, was on many occasions observed as a striking pearl-like object when on or very near the edge of the disc; but when far inside the border or near central region the contrast was not nearly so strong. The intensification of the bright spots when on the margin of the disc has sometimes probably occasioned their recognition as "projections." The appearance alluded to is by no means a new feature. The bright spot described above as situated east of the *Mare Acidalium* was frequently seen by

Mr. E. B. Knobel in 1873 May, and its position indicated in a number of sketches (*Monthly Notices*, vol. xxxiii. p. 476). Mr. Knobel says : "I thought the spot brighter on the terminator than when near the centre of the disc. The sketch made May 22 represents the white spot apparently raised above the disc by irradiation." Mr. N. E. Green also says : "One of the most marked features of the opposition of *Mars* now past has been the frequent appearance of masses of light on the limb which never arrived at the meridian" (*Monthly Notices*, vol. xlvi. p. 447, 1886 June).

With regard to the question of real or supposed changes in Martian detail we must obviously be prepared to make large allowances for variations in the character of the seeing, and it should be remembered that the observer himself is liable to certain perturbations which may materially affect his results. But while fully considering such influences I believe that very palpable alterations occur in the aspect of some of the features, though such variations are only temporary and due to atmospheric causes.

Some writers (including the Rev. E. Ledger, who has a paper on the subject in the *Nineteenth Century* for May) apparently regard the objective existence of the canals as still open to question, but there is really no doubt whatever about the streaked or striated configuration of the northern hemisphere of *Mars*. The canals do not appear as narrow straight deep lines in my telescope, but as soft streams of dusky material with frequent condensations. This statement holds good as regards the usual appearance of the more delicate features. But on a few occasions the canals, where they join the seas just south of the equator and cross the bright areas to be found there, were outlined very sharply. For instance on May 9 and 12 the canals running north from the region of the "Forked Bay" appeared under this aspect :—

The double canals are an optical effect only, though in a few cases two canals may lie near together and run almost parallel ; their contiguous situation appears, however, to be the result of accident. The surface lineaments generally are as plentiful as they are irregular and unsymmetrical, and their aspect is far more highly suggestive of natural than artificial production.

Jupiter was observed on the mornings of May 27, June 1, 4, 5, 6, and 7. A few selected transits were as follows :—

		h m	Long. °	System.
Great Red Spot	May 26	16 18	29·5	II.
„ „	May 31	15 28	30·4	„
„ „	June 5	14 33	28·3	„
P. end dark mass, S. tropical zone	June 6	15 16	204·5	„
Very bright equa. spots	June 3	15 19	268·1	I.
	June 6	14 49	3·4	„
Very dark equa. spot	June 3	15 0	256·5	„
Dark N. tropical spots	June 3	14 45	95·1	II.
	June 3	15 28	121·0	„
	June 3	16 1	141·0	„
Dark N. temp. spot	May 26	15 38	5·3	„
	May 31	14 50	7·4	„
Dark N.N. temp. spot	June 6	15 12	202·1	„

The accelerated motion of the Great Red Spot has been continued for this object, has lost about 8° in longitude since 1902 December (when it was 37°), and this corresponds to a rotation period of $9^h\ 55^m\ 38^s{\cdot}8$. The large dark spot in the S. tropical zone, frequently observed in 1901 and 1902, is still prominently visible, and its p. side will be in longitude 185° at the middle of 1903 July.

In the various observations here described a 10-inch reflector was employed with a power of 312, but occasionally on *Mars* magnifiers up to 488 were found serviceable.

Bishopston, Bristol: 1903 *June* 7.

Postscript.—From my subsequent observations of the Great Red Spot on *Jupiter*, including one obtained on July 13, $16^h\ 2^m$, when the object was in longitude 32°·8, it appears certain either that the motion has been recently retarded or that my transits at the end of May and beginning of June were a little too early.

July 14.

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1902 *November* 12 *and* 1903 *April* 27.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of *Neptune's* satellite were made from photographs taken with the 26-inch refractor of the Thompson Equatorial. The occulting shutter was used when not otherwise noted, and the position of

the plate adjusted so that the planet was behind the shutter while the satellite was just outside it. In one or two cases the planet was not put sufficiently far behind the shutter so that the image falls very near its edge and is not clearly separated from the outer halo. These are noted as "partly occulted."

By an arrangement of Mr. Davidson's the occulting shutter has been made to work automatically, so as to give an exposure of about $\frac{1}{10}^{s}$ every 20^{s} to the planet, the satellite being meanwhile exposed for from 20^{m} to 30^{m}.

The zero of position-angle was obtained by stopping the clock and giving a short supplementary exposure. Generally, several short exposures were given, the clock being stopped for a short time between each of them.

Owing to wind on some nights and bad definition on others, this series of photographs is not quite so good as that taken between 1902 January 6 and 1902 April 10.

The photographs were taken by Messrs. Davidson, Edney, or Melotte, and the measurement of the photographs was made by Mr. Edney and Mr. Burkett, in the manner described in *Monthly Notices*, vol. lxii. p. 623. A mean correction for refraction has been applied.

The tabular positions were computed from the data given in the *Connaissance des Temps* based on Dr. H. Struve's elements, the eccentricity of the orbit being neglected owing to the uncertainty as to the present position of the periastron.

Neptune and Satellite.

Position-angle and Distance, from Photographs taken with the 26-inch Refractor.

Date and G.M.T.				Position Angle. Observed.	Position Angle. Tabular.	O–C.	Distance. Observed.	Distance. Tabular.	O–C.	Remarks.
1902.	d.	h.	m. s.	°	°	°	″	″	″	
Nov.	12	11	44 44	60·40	59·51	−0·89	15·04	14·85	−·19	Not occulted.
	12	12	21 2	58·56	58·21	−0·35	14·19	14·71	+·52	Not occulted.
	12	13	34 26	55·57	55·53	−0·04	14·45	14·41	−·04	Not occulted.
	12	14	8 35	55·34	54·24	−1·10	14·14	14·26	+·12	Not occulted.
	13	11	37 32	345·61	342·98	−2·63	10·89	11·03	+·14	
	13	12	3 44	340·65	341·31	+0·66	10·85	11·08	+·23	
	17	11	34 29	95·16	96·87	+1·71	16·71	16·40	−·31	
	20	11	32 53	274·09	274·35	+0·26	—	—		Planet partly occulted.
	20	12	14 44	272·24	273·16	+0·92	—	—		Planet partly occulted.
	28	12	2 51	135·78	136·10	+0·32	12·70	12·48	−·22	
	28	12	35 27	135·21	134·48	−0·73	12·70	12·62	−·08	Satellite faint.
Dec.	29	7	41 34	68·66	67·83	−0·83	15·78	16·03	+·25	Satellite very faint.
	29	8	17 4	66·84	66·73	−0·11	15·60	15·92	+·32	Satellite faint.

Date and G.M.T.				Position Angle. Observed.	Tabular.	T-O.	Distance. Observed.	Tabular.	T-O.	Remarks.
02. d.	h.	m.	s.	°	°	°	″	″	″	
31	10	17	26	281·42	281·98	+0·56	15·73	15·94	+·21	
31	10	53	20	280·98	280·88	−0·10	15·88	16·05	+·17	
03.										
an. 1	12	49	30	234·64	234·52	−0·12	14·57	14·59	+·02	
3	9	14	8	98·43	101·18	+2·75	15·68	16·01	+·33	Very faint.
3	9	47	52	98·28	100·15	+1·87	16·04	16·10	+·06	
15	9	40	9	89·37	91·54	+2·17	17·21	16·75	−·46	
15	10	13	32	89·09	89·05	−0·04	16·64	16·79	+·15	
15	10	55	44	87·16	87·88	+0·72	16·52	16·82	+·30	
23	9	3	23	311·49	313·17	+1·68	12·67	12·52	−·15	
23	9	32	6	309·83	311·76	+1·93	12·59	12·64	+·05	
23	10	4	28	307·03	310·19	+3·16	12·67	12·78	+·11	Planet partly occulted.
25	9	1	42	215·28	214·01	−1·27	12·69	12·50	−·19	Satellite faint.
28	7	40	55	33·63	33·64	+0·01	12·20	12·47	+·27	
28	8	13	12	29·36	32·01	+2·65	11·84	12·33	+·49	
eb. 1	9	40	25	116·37	119·57	+3·20	13·90	13·77	−·13	Satellite very faint.
2	8	5	51	77·00	77·77	+0·77	16·67	16·68	+·01	
6	9	4	8	191·21	194·33	+3·12	11·36	11·25	−·11	Satellite very faint.
10	8	45	32	290·70	291·47	+0·77	15·26	14·59	−·67	Satellite very faint.
10	9	22	25	289·89	290·15	+0·26	15·23	14·74	−·49	
16	11	29	9	278·93	280·16	+1·23	15·57	15·76	+·19	
16	11	54	33	279·26	279·38	+0·12	15·56	15·83	+·27	
17	7	32	13	246·67	245·83	−0·84	15·48	15·73	+·25	Satellite very faint.
17	8	2	41	244·06	244·89	+0·83	15·61	15·64	+·03	Satellite very faint.
17	8	53	7	243·87	243·30	−0·57	15·17	15·48	+·31	
18	9	45	50	167·75	169·00	+1·25	10·95	10·76	−·19	
18	10	13	4	166·28	167·21	+0·93	10·71	10·72	+·01	
18	10	39	23	165·23	165·49	+0·26	10·64	10·79	+·15	Satellite ill-defined.
23	7	26	6	239·34	240·37	+1·03	15·15	15·13	−·02	Satellite very faint.
26	8	11	58	54·97	55·80	+0·83	14·43	14·59	+·16	
28	8	45	50	273·88	274·44	+0·56	16·17	16·14	−·03	
28	9	11	41	272·80	273·69	+0·89	15·86	16·19	+·33	
28	9	38	16	271·28	272·93	+1·65	15·65	16·24	+·59	
Iar. 3	7	20	8	92·28	94·38	+2·10	16·31	16·11	−·20	Satellite very faint.
3	7	59	50	93·97	93·22	−0·75	15·77	16·19	+·42	
3	9	56	42	91·53	89·91	−1·62	16·31	16·38	+·07	Satellite very faint.
6	8	50	4	269·23	269·30	+0·07	16·01	16·38	+·37	
6	9	15	38	267·32	268·59	+1·27	16·14	16·42	+·28	
6	9	43	32	266·89	267·82	+0·93	16·53	16·44	−·09	

Date and G.M.T.				Position Angle. Observed.	Tabular.	T-O.	Distance. Observed.	Tabular.	T-O.	Remarks.
1903.	d.	h.	m. s.	°	°	°	″	″	″	
	11	8	33 45	315·87	316·18	+0·31	11·67	11·94	+·27	
	11	9	7 2	314·72	314·46	−0·26	11·79	12·07	+·28	
	13	8	24 15	216 73	217·26	+0·53	12·35	12·56	+·21	Satellite faint.
	15	8	32 36	81·94	82·48	+0 54	16·33	16·44	+·11	
	15	9	14 32	80·24	81·33	+1·09	16·19	16·44	+·25	
	16	10	19 44	26·92	26·98	+0·06	11·90	11·72	−·18	
	16	10	46 43	25·58	25·51	−0·07	11·73	11·62	−·11	
	21	8	38 54	77·70	77·45	−0·25	15·84	16·28	+·44	
	21	9	7 50	77·16	76 65	−0·51	15·57	16·25	+·68	Satellite faint.
	26	9	19 19	112·97	114·45	+1·48	13·72	13·88	+·16	
Apr.	14	7	59 40	55·34	57·23	+1·89	14·38	14·34	−·04	Planet partly occulted.
	16	8	17 11	274·31	276·19	+1·88	15·63	15·61	−·02	
	17	8	22 2	233·39	233·24	−0·15	14·03	13 88	−·15	
	24	8	39 47	141·01	141·93	+0·92	11·33	11·32	−·01	
	27	8	39 49	316·08	317·16	+1·08	11·80	11·63	−·17	

The Great Nebula in Auriga. By Dr. Max Wolf.

In *Astr. Nachr.* 3130 (1892 October 9) I gave a description of some recently discovered nebulæ in *Auriga*. The most remarkable of these was discovered independently by Schaeberle, E. von Gothard, and myself near the 6·7 mag. star B.D. + 34°, 980. This nebula has received the number 405 in Dreyer's Index Catalogue. Two other nebulæ then discovered were given by Dreyer the numbers 410 and 417; to the east of 417 lies the nebula G.C. 1137.

When photographing the region of Nova *Aurigæ* with the two 16-inch Brashear lenses on the evening of 1902 March 6 with five hours' exposure I found a large diffused nebulosity at the edge of the plates near the places of the above-named nebulæ. In the present year, on 1903 February 19, I took a further photograph with the region in the centre of the field of my two 16-inch lenses.

The plates were exposed from $7^h\ 9^m$ to $12^h\ 14^m$ Koenigstuhl M.T. They show such interesting nebulosities that I send the accompanying pictures to the Society.

The reproduction (plate 20) is an untouched contact print from one of the original plates. A great part of the plate is covered with nebulosity, condensed around five bright groups,

KEY MAP.

NEBULOSITIES IN AURIGA.

NEBULOSITIES IN AURIGA.

PHOTOGRAPH BY DR. MAX WOLF, HEIDELBERG.

which form a great V. It looks like a photograph of the V of *Taurus*, the corner shifted somewhat westward.

The five groups are as shown on the key map (proceeding from east to west); first, the splendid cluster, Messier 38 and its accompanying cluster, Herschel VII. 39; second, the group near *φ Aurigæ*; third, the great nebulæ Ind. Cat. 410; then the group with the bright stars near 16 and 19 *Aurigæ*; and finally the remarkable nebula Ind. Cat. 405 connected with the B.D. star 34°, 980. Herschel's G.C. 1137 between Ind. Cat. 417 and the star 36 *Aurigæ* is involved in the same luminous matter.

It was in 1892 (September 25 and 30) that I first photographed the nebulæ Ind. Cat. 417, 410, and 405. My present intention is to show the *connection* between all these nebulæ, star groups, and clusters. It will be seen from the plate and key map that the above-named five or six groups are involved in the same nebulosity. This appears to support the theory that nearly all these stars, clusters, and nebulæ are at the same order of distance from our Earth as in similar cases in *Cygnus* and *Orion*.

The nebulæ G.C. 1137, I.C. 417 and 410 show many interesting features, but are much surpassed by the nebula Ind. Cat. 405. The more distant exterior parts of this nebula are of interest as showing the *lacuna-forming* forces of the nebulæ: the north and north-western borders form the boundaries of a great lacuna in which the fainter stars seem to have disappeared. The same is the case with the western borders of the branch A (see key map). The drift is here always from west or north-west towards east. This matter deserves further investigation by careful counts of the number of stars in the different regions.

I only wish to direct attention to the remarkable inner structure of the nebula I.C. 405. My second photograph, of which I send short and long printed pictures,* is enlarged about five times from the original plate without retouching.

The long printed picture shows the interior parts and the disc of the star; on the shorter printed one the disc of the star flows into the surrounding nebulæ, but the more distant nebulæ are quite visible. The large star-disc is B.D. 34°, 980. It looks like a burning body from which several enormous curved flames seem to break out like gigantic prominences. One of these seems to connect B.D. 34°, 980 with the 7·8-magnitude star B.D. 34°, 978.

It seems to me that it would be interesting to examine this "*flaming star*" with the spectroscope, and I hope that my pictures may draw the attention of spectroscopists to this star, which seems physically connected with the nebulous matter.

Koenigstuhl Astrophysical Observatory:
1903 June.

* The detailed structure of the nebula I.C. 405 is, however, better shown in the photograph published by Dr. Isaac Roberts in *Knowledge* for 1903 April. The editors of the *Monthly Notices* have therefore not considered it necessary to reproduce Dr. Wolf's enlarged photographs.

Further Observations of the New Star in Auriga, with the Mean Magnitudes for the years 1892–1903, from estimations made at the Radcliffe Observatory, Oxford.

(*Communicated by Arthur A. Rambaut, M.A., D.Sc., F.R.S., Radcliffe Observer.*)

The observations of Nova *Aurigæ* given in this paper are in continuation of those published in the *Monthly Notices*, vol. lxi. p. 543. Estimations of the magnitude of this star have also appeared in vol. lii. pp. 430–1, vol. liii. pp. 33, 34, 126, vol. lv. p. 164, vol. lvi. p. 234, vol. lviii. p. 180, and vol. lix. p. 258.

A further decrease of brightness was noted at the beginning of this year, when the object proved a very difficult one with the 10-inch Barclay telescope.

1902 Dec. 31. $11^h\ 45^m$ G.M.T. Barclay Equatorial, power 90.

Nova fainter than any of the comparison stars in the chart (*Monthly Notices*, vol. lii. p. 431).

Nova = c' near c, or M of Burnham's list (*Monthly Notices*, vol. lii. p. 435). M was estimated on 1900 Nov. 17 in the Barclay 0·2 mag. fainter than p (*Monthly Notices*, vol. lxi. p. 543).

Nova visible in glimpses, and seen best by averted vision (R.).

1903 Jan. 2. 11^h G.M.T. Barclay Equatorial, power 85.

Nova fainter than any of the comparison stars in the chart, but equal to Burnham's M, or c' near c (Radcliffe chart). (R.)

1903 Jan. 2. $11^h\ 10^m$ G.M.T. Power 180.

Nova distinctly seen and equal to Burnham's M, or c', but slightly fainter than p. With this power the star close to the Nova (designated in Burnham's list as B) is just visible, and is very nearly at the limit of vision with the Barclay telescope. The Nova is approximately a magnitude brighter than this star (R.).

The resulting magnitudes of Nova *Aurigæ* are :

	Radcliffe Scale.*	Burnham's Scale.†	Barnard's Scale.‡
1902 Dec. 31	14·5	13·2	12·2
1903 Jan. 2	14·5	13·2	12·2

Observer : R., Mr. Robinson.

* *Monthly Notices*, vol. lii. p. 431.

† *Monthly Notices*, vol. lii. pp. 433–6.

‡ *Monthly Notices*, vol. lxii. p. 65.

Erratum.—Monthly Notices, vol. lxi. p. 543, sixth line from bottom : *For* between *read* beyond.

The mean magnitudes from 1892 to 1903 are:

1892	4·5 to <14·0, Feb. 3 to March 31.
1892	9·5 to 9·8, Sept. 8 to end of year.
1893	9·7
1894	9·7
1895	9·7
1896	—
1897	11·4
1898	12·0
1899	13·4
1900	13·9
1901	—
1902–3	14·5

Radcliffe Observatory, Oxford:
1903 *June* 11.

Further Observations of the New Star in Perseus made at the Radcliffe Observatory, Oxford.

(*Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.*)

This paper is in continuation of the notes on the same subject published in the *Monthly Notices* for 1901 March, April, May, June, November, and 1902 June.

The Nova is gradually diminishing in brightness, but more slowly than before. During 1902 the rate of diminution was about $0^{M}\cdot004$ or $0^{M}\cdot005$ per diem.

The colour was still bluish on September 3.

The magnitudes of the last five comparison stars in the list are those of Hagen's *Second Chart and Catalogue for Observing Nova Persei.*

Table I.

List of Stars used for comparison with Nova Persei.

Ref. No.	Name of Star.	Adopted Tabular Magnitude.	Authority for Magnitude.
77	Arg. Z. +43° 739	9·0	Argelander D.M.
81	Arg. Z. +43, 744	8·6	,, ,,
82	Arg. Z. +43, 746	9·1	,, ,,
83	Arg. Z. +43, 751	9·0	,, ,,
84	Arg. Z. +43, 749	9·0	,, ,,

Ref. No.	Name of Star.	Adopted Tabular Magnitude.	Authority for Magnitude.
85	Arg. Z. +43°, 743	9·4	Hagen (Chart II.).
86	Hagen II. 42	10·1	,, ,,
87	Arg. Z. +43, 738	9·7	,, ,,
88	Arg. Z. +43, 737	9·8	,, ,,
89	Hagen II. 44	10·3	,, ,,

TABLE II.

Means of Estimations of Magnitude of Nova Persei.

1902.	G.M.T. h m	Observer.	Aperture of Telescope. Inch.	Power used.	Reference Stars.	Mean Mag. of Nova Persei.
Sept. 3	13 30	R.	10·0	90	77, 85, 86	9·35
5	11 0	W.	,,	,,	77, 86	9·50
6	10 45	R.	,,	,,	81, 82, 83, 84, 77, 85, 87, 88, 80, 89	9·38
Dec. 31	11 20	R.	,,	,,	77, 85, 86	9·93

Observers' Remarks.

1902.
Sept. 3. The image of the Nova is dull and bluish. The comparison star No. 86 has a very faint companion following (R.).

Observers : W., Mr. Wickham ; R., Mr. Robinson.

Radcliffe Observatory, Oxford :
1903 *June* 11.

Observations of the New Star in Gemini made at the Radcliffe Observatory, Oxford.

(*Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.*)

On March 25 we received from Professor Turner an announcement of his discovery of a new star in *Gemini*. The first opportunity of observing the object occurred on March 26, and since then observations at intervals have been made of its brightness.

The star has generally presented a red or reddish appearance ; but observers' notes seem to suggest the probability that there is a slight change taking place in colour in the direction of diminishing redness.

The observations show a decline in the brightness of the star at an average rate of about $0^{M}\cdot015$ per diem. The diminution is not quite uniform, but the observations are not sufficiently

numerous to enable us to decide whether the variations are real or apparent.

The magnitudes of the comparison stars Nos. 1, 3, and 4 are taken direct from the Harvard Photometric Durchmusterung; those of Nos. 2, 5, 9, and 11 are the Harvard values as given in the "Notes" column of Hagen's *Chart and Catalogue for Observing Nova Geminorum*; while those of the remaining stars of the list are based on Radcliffe estimations on the Harvard scale.

TABLE I.

List of Stars used for comparison with Nova Geminorum.

Ref. No.	Name of Star.	Adopted Tabular Magnitude.	Authority for Magnitude.
1	Arg. Z. +29° 1342	8·16	Harvard Photom. D.M.
2	Arg. Z. +30, 1320	8·53	Harvard (Hagen's Chart).
3	Arg. Z. +30, 1306	8·76	Harvard Photom. D.M.
4	Arg. Z. +30, 1332	8·01	„ „ „
5	Arg. Z. +30, 1316	9·26	Harvard (Hagen's Chart).
6	Hagen 30	9·66*	Radcliffe observations, based on Harvard.
7	Arg. Z. +30, 1321	9·56	„ „ „ „
8	Arg. Z. +29, 1328	9·56	„ „ „ „
9	Arg. Z. +30, 1309	10·13	Harvard (Hagen's Chart).
10	Arg. Z. +30, 1315	(10·22)	Radcliffe observations, based on Harvard.
11	Arg. Z. +30, 1317	10·08	Harvard (Hagen's Chart).

TABLE II.

Means of Estimations of Magnitude of Nova Geminorum.

1903.	G.M.T. h m	Observer.	Aperture of Telescope. Inch.	Power used.	Reference Stars.	Mean Mag. of Nova Geminorum.
March 26	8 10	A.A.R.	10·0	45	1, 2	8·70
	8 10	W.	„	„	1, 2	8·62
	10 0	R.	„	„	1, 3, 2	8·58
	10 0	C.	„	„	1, 3, 2	8·54
30	12 15	R.	7·5	„	1, 3, 2	8·61
April 4	9 45	R.	10·0	„	4, 1, 3, 2	8·67
13	8 30	R.	„	„	3, 2, 5, 6	9·13
20	11 0	R.	„	„	3, 2, 5, 6	9·13
29	9 30	W.	„	„	—	10· ±
May 4	9 45	R.	„	„	1, 2, 5, 6, 9	9·45
8	9 0	W.	„	„	5, 6	< 9·66
20	10 40	R.	„	„	1, 3, 2, 5, 7, 8, 6, 11, 9, 10	9·42

* No. 6. Wide double. Combined light observed.

Observers' Remarks.

1903.
March 26. Nova is reddish-yellow (W.). Nova distinctly red (R.). Nova reddish (C.).
March 30. Observed with the Heliometer. The magnitude of Nova and comparison stars are sensibly the same as those observed with the Barclay on March 26 (R.).
April 4. Nova reddish, not so red as on March 26, but moonlight strong to-night (R.).
April 13. Nova red (R.).
April 29. Tried to estimate magnitude of Nova, but clouds too frequent. I should think it is as low as 10 mag., but night very unfavourable (W.).
May 4. Sky hazy, moonlight strong (R.).
May 8. Nova fainter than Nos. 5 and 6. I could not see No. 6 as a double, and the smaller stars were obliterated by increasing haze. Sky quite thick at 9.30 G.M.T. (W.)
May 20. Stars very low. Observations of the two faintest stars (Nos. 9 and 10) difficult (R.).
June 3. Nova looked for at 10^h G.M.T., but was not visible. Twilight too strong, and altitude of object small (R.).

The observers were :—

Dr. Rambaut, indicated by	...	...	...	A.A.R.
Mr. Wickham,	,,	...	...	W.
Mr. Robinson,	,,	...	...	R.
Mr. McClellan,	,,	...	...	C.

Radcliffe Observatory, Oxford :
1903 *June* 11.

Positions of 166 *Stars around Nova Geminorum ; with a Discussion of Systematic Differences between two Exposures on the same Plate.* By F. A. Bellamy.

In a paper (*Monthly Notices*, vol. lxiii. p. 326) I gave the magnitudes and positions of Nova *Geminorum* and fifteen stars near it. The plate from which those results were derived did not show many stars fainter than the 10·3 magnitude. Since that date several plates of the region have been obtained, among them being the two upon which this paper is based :

Centre.
Plate 2222, $+30°$, $6^h\ 36^m$, exposed 1903 April 17 for 30 min. and 35 min.
,, 2240 ,, ,, ,, ,, ,, 22 ,, 100 min.

As this portion of the sky was considerably towards the western horizon when darkness commenced the mean hour-angles for these three exposures were $3^h\ 55^m$, $4^h\ 24^m$, and $5^h\ 2^m$ respectively.

On plate 2222 125 stars have been measured in a square with sides equal to 30′ surrounding the Nova, which is approximately 8^{0s} to the east and $2\frac{1}{2}'$ north of the plate centre; from this number it may be estimated that there are 2,400 stars on the whole plate. On plate 2240, with 100 minutes' exposure, only four réseau squares, equal to a square with sides of 10′, with the Nova in the centre, have been used, and within this area 57 stars have been measured, from which it may be estimated that there are certainly not less than 10,000 stars on the whole plate. On both these nights the sky was clear, but the hour-angles at which the three exposures were made would tend to reduce the number of the faintest stars that would have been recorded if the plate had been exposed on the meridian.

Plate 2222.

This plate covers the region in which the positions of 51 stars have been recorded in the Cambridge and Leiden A.G. Catalogues. These positions were brought up to epoch 1900·0, with precessions (given in the catalogues), but no corrections for proper motion, magnitude equation, or systematic differences between the catalogues have been applied, and the places for 1900·0 were then converted into standard coordinates in réseau intervals as unit for plate centre R.A. $6^h\ 36^m$, Decl. $+30°$. If (ξ, η) denote standard coordinates referred to the plate centre as origin $\xi' = \xi + 13$, $\eta' = \eta + 13$, so that (ξ', η') are referred to the corner of the réseau and are always positive, and (x, y) denote measures on the plate expressed in réseau intervals, each measure being the mean of two in direct and reversed positions of the plate. Then putting

$$ax + by + c = x - \xi$$
$$dx + ey + f = y - \eta$$

the values of a, b, c, d, e, f were separately determined for each exposure as follows:

	a	b	c	d	e	f
Exposure I.	−·00057	−·00003	+·0586	−·00030	−·00068	−·0258
,, II.	−·00066	+·00005	+·0635	−·00049	−·00075	−·1713

The differences in the constants c and f are due to the displacement of the telescope in order to get the two exposures sufficiently separated on the plate; the differences in the other constants are partly due to refraction effects and partly accidental.

The effect of differential refraction (see *Monthly Notices*, vol. liv. p. 18) is to *diminish* the coordinates by the quantities

$$\beta_0(1 + X^2)x + \beta_0 XY y$$
$$\beta_0 XY x + \beta_0(1 + Y^2)y$$

where XY are the coordinates of the zenith on the plate expressed in circular measure and β_0 is the coefficient of refraction. Adopting $\beta_0 = 1/3600 = \cdot000278$, the numerical values of these are

Exposure I. ... (H.A. $3^h\ 55^m$) $+\cdot00045x + \cdot00017y$; $+\cdot00017x + \cdot00044y$
„ II. ... (H.A. $4^h\ 24^m$) $+\cdot00052x + \cdot00023y$; $+\cdot00023x + \cdot00050y$

and if we subtract this effect of refraction the constants given above become

		a'	b'	d'	e'
Exposure I.	...	−·00012	+·00014	−·00013	−·00024
„ II.	...	−·00014	+·00028	−·00026	−·00025

which are in satisfactory accordance, if we admit a possible change of orientation of ·00014 between the exposures. [It may be remarked that no special care has been recently exercised in adjusting the polar axis, as for short exposures no such care is necessary.]

When the measures of individual stars are corrected by the above formulæ and compared with the calculated standard coordinates residuals are found, as in the following Table I. The agreement between the two exposures is satisfactory generally; in one case only is there a difference of ·005 or 1″·5; in three cases of ·004 or 1″·2; all other differences are less than 1″·0.

It may be remarked that 9·5 magnitude stars give discs about 7″ in diameter with these long exposures; thirty of the fifty-one stars measured have diameters over 12″, and some diameters exceed 20″, and it is not easy to bisect accurately these large images.

TABLE I.

Residuals for the Reference Stars.

No. in Camb. or Leiden.	1st Exp. x.	2nd Exp. x.	1st Exp. y.	2nd Exp. y.	No. in Camb. or Leiden.	1st Exp. x.	2nd Exp. x.	1st Exp. y.	2nd Exp. y.
3365	+002	+003	+001	−002	3443	+004	+006	000	+004
3369	−001	−001	+003	+001	3446	−004	−002	+004	+002
3371	+003	000	+003	+003	L 2782	+005	+002	−002	−003
3374	−006	−007	−003	+001	3447	−010	−010	+003	+001
L 2733	−005	−005	−001	−001	L 2784	+002	+001	−001	+002
L 2735	+005	+002	−001	−004	L 2786	+002	+001	−001	−002
3377	−001	−001	−002	−001	L 2789	−002	−002	+001	+001
L 2737	+004	+005	−004	−005	3455	−004	−003	000	000
3381	−003	000	+002	000	L 2793	+003	+003	−003	−003
L 2742	+002	+002	+003	+003	L 2794	+002	+002	+005	+003
3388	−008	−006	+002	+003	L 2797	+001	−001	+002	−002
3390	−001	+001	+003	+002	3465	+003	+001	−003	000

No. in Camb. or Leiden.	1st Exp. x_c.	2nd Exp. x_c.	1st Exp. y_c.	2nd Exp. y_c.	No. in Camb. or Leiden.	1st Exp. x_c.	2nd Exp. x_c.	1st Exp. y_c.	2nd Exp. y_c.
L 2754	+001	−001	−002	−001	3467	−013	−013	000	+001
3400	−002	−001	+006	+003	L 2799	+002	000	000	−002
3404	−002	−004	−001	−004	L 2800	−001	000	+001	−002
3406	−003	−002	+001	+003	3468	−005	−005	−001	−001
3407	+001	+001	+002	+004	3469	+001	000	+002	+004
3409	−009	−008	−010	−005	L 2803	+001	+003	−001	−001
L 2763	+003	+003	−001	−002	3471	−004	−004	−003	−003
3424	−001	−001	−003	−004	3473	+002	+002	000	−001
3425	+003	+003	+002	+001	L 2805	000	−002	−003	−003
3426	+002	+003	000	+002	3477	+003	+004	−001	−001
3430	000	−001	+007	+008	3482	−002	−001	+002	+003
L 2773	+004	+003	000	000	L 2815	−003	−001	000	000
3439	+003	+005	000	−001	3486	−004	−002	−003	−005
3441	+004	+002	−001	+001					

Besides measuring these fifty-one stars I also measured on this plate 125 stars near the Nova. These measures were corrected by the formulæ derived above and the standard coordinates deduced.

Before combining the two exposures the differences between them were formed, and it was found that the mean differences were

$x_I - x_{II}$	$y_I - y_{II}$
+ ·00074 or + 0″·22	− ·00044 or − 0″·13

The mean of the differences without regard to sign is ± ·0013 or ± 0″·4 in both x and y, so that the above means are too large to be entirely accidental. The constants deduced from the Cambridge and Leiden stars do not seem entirely to suit the faint stars near the Nova, and the cause may be sought either in the difference of brightness or in the locality ; and, further, it may be of such a kind as to be common to different measurers, or to vary with the measurer.

To elucidate the matter the 125 stars were first measured again by Mr. B. G. C. Gray, and it was found that our mean differences were

	x B−G	y B−G
Exposure I.	·0000	·0000
Exposure II.	−·0003	+·0001

The next step taken was to arrange the differences according to diameter of the star image as in Table II.

TABLE II.

Differences between exposures (I.–II.) *arranged according to diameter of star image.*

(The unit is 0″·3 or ·001 of a réseau interval.)

Diameter of image.	No. of stars.	*x* Difference.		*y* Difference.	
		Bellamy.	Gray.	Bellamy.	Gray.
50 to 75	10	+0·1	...	+0·4	...
40 „ 49	20	+0·2	...	+0·1	...
30 „ 39	17	−0·4	...	−0·9	...
30 „ 39	7	−0·64	−0·57	−0·21	+0·43
20 „ 29	18	+0·67	+0·56	−0·12	−0·18
15 „ 19	22	+0·43	+0·35	−0·42	−0·12
11 „ 14	34	+0·75	+0·32	−0·35	−0·23
8 „ 10	32	+1·06	+0·20	−0·52	−0·36
7 and less	13	+1·30	+0·83	−1·07	−0·77

The first three groups refer to forty-seven of the fifty-one stars—the other four stars are below 30 in diameter—from the Cambridge and Leiden Catalogues; but the work was only carried to ·001 of a réseau interval, and Mr. Gray did not measure these stars. Remembering that the measures are only estimated to a unit of the above table, and therefore allowing for considerable accidental error, especially in the smaller groups, we may conclude that the table gives satisfactory evidence of a small systematic difference depending on the magnitude of the star, which may be due to the measurer or may be an outcome of the irregularities in keeping the guiding star bisected accurately during the exposure. Indeed an effect of this kind is theoretically probable in all cases; it is most unlikely that the unavoidable excursions of the guiding star will be precisely symmetrical in all directions, and any uncompensated excursion may elongate the image of a bright star slightly without affecting the image of a faint star.*

It is not possible to follow up the investigation of this difference further at present; and we must be content to take the mean of the two exposures. In Table VI. each coordinate of these 126 stars is obtained from eight measures in all; the Nova from twenty measures (two measures, two exposures, and two positions of the plate).

But since this plate does not show stars beyond about the 13·5 magnitude there have also been measured (and the

* It seems probable that this error is identical with that mentioned as a "guiding error" on p. 30 of Publication No. 10 of the Astronomical Laboratory at Groningen, just received. But we have not yet received Publication No. 1 in which this error is defined.

measures incorporated in Table VI.), forty-one fainter stars only shown on the plate with 100^m exposure, as will now be described.

Plate 2240.

Exposed for 100^m on 1903 April 22. In the small square of side 10′ surrounding the Nova fifty-seven stars were measured, forty-one of which are not visible for measurement on plate 2222. Of special interest are the six stars measured by Professor E. E. Barnard (*Yerkes Obs. Bulletin*, No. 19, or *Astroph. Journ.* xvii. p. 302), reference to which may be facilitated by the following table :—

TABLE III.

Barnard's (Y.O.B. No. 19) Ref. No.	Barnard's (Y.O.B. No. 19) Magnitude.	Pickering's (Harv. Circ. No. 70) Ref. No.	Pickering's (Harv. Circ. No. 70) Magnitude.	Oxford (TABLE VI.) Ref. No.	Oxford (TABLE VI.) Diameter on plate 2240.	Inferred magnitude.
1	12·3	γ	13·53	83	16	13·5
2	12·6	—	—	80	10	14·5
3	14·5	—	—	75	4	15·5
4	12·5	β	13·35	74	18	13·0
5	12·5	—	—	68	19	12·8
6	12·0	α	13·05	82	18	13·0

Pickering's three stars α, β, γ are visible on plate 2222. If we assume that Barnard's estimates are numerically 1 magnitude too small the faint star 3 (Oxford 75), which is just visible on plate 2240, is of about magnitude 15·5. The star 2, estimated by Barnard as equally bright with 1 and 4, is photographically fainter by a magnitude or more.

Measures of the Nova on plate 2240 were made by Mr. H. C. Plummer, Mr. E. A. Gray, and myself, and we all used several methods, bisecting the image, or taking contact observations of its perimeter &c. The fifty-seven faint stars near the Nova were measured by myself alone. For the determination of the constants of plate 2240 twelve stars were selected within 15′ of the Nova, for which standard coordinates had been deduced from the measures of the first exposure of plate 2222. [The measures of the second exposure were not at this time available.] Comparison of the measures of these twelve on plate 2240 with their standard coordinates gave the following plate constants :—

	a_1	b_1	c_1	d_1	e_1	f_1
Solution 1	−·00078	−·00198	+·1405	+·00122	−·00108	−·0389

These constants derived from stars in a restricted area on the plate should give satisfactory results for stars within the area, and probably better results than the bright stars previously used for plate 2222, the discs of which were larger than ever. But as a check on the constants four pairs of stars about 35′ from the

plate centre were also selected for measurement, and the constants thus independently deduced were

	a_2	b_2	c_2	d_2	e_2	f_2
Solution 2	−·00072	−·00205	+·1391	+·00139	−·00094	−·0428

A comparison of the residuals from constants 1 and constants 2 is given in Table IV.

TABLE IV.

Plate 2240–2222.

Standard Coordinates.		Residuals in x Solution.		Residuals in y Solution.	
ξ_1	η_1	(1)	(2)	(1)	(2)
14·6456	10·7925	+·0007	+·0019	−·0009	−·0011
14·9803	10·8371	+ 14	+ 25	+ 11	+ 10
15·6151	10·6177	− 18	− 7	− 11	− 12
19·8740	10·6610	− 9	0	0	− 9
20·4223	11·1470	− 3	+ 7	+ 4	− 8
20·5139	11·0765	+ 6	+ 16	+ 7	− 4
20·7052	15·9783	− 4	+ 8	− 1	− 19
19·5020	16·2573	+ 4	+ 17	+ 2	− 18
20·5336	16·3616	+ 3	+ 16	− 8	− 26
16·0666	16·5058	+ 1	+ 17	+ 17	+ 5
14·9455	16·0547	+ 11	+ 27	− 12	− 21
15·0960	15·8960	− 18	− 3	+ 5	− 4
Mean . .		·0000	+ ·0012	·0000	− ·0010
6·4020	13·1041	−·0020	−·0001	−·0005	+·0005
5·8247	12·1288	− 11	+ 8	− 9	+ 1
19·8401	12·4597	− 10	+ 1	+ 30	+ 16
19·4214	12·5416	− 4	+ 6	+ 1	− 11
12·6467	20·3089	− 28	− 6	+ 20	+ 8
13·8514	20·6995	− 22	− 2	− 1	− 17
13·3988	7·3208	− 9	+ 2	− 3	+ 1
14·1247	7·0919	− 21	− 11	− 15	− 13
Mean . .		−·0016	·0000	+·0002	−·0001

No one would probably think of using solution 1 derived from the twelve stars to obtain places of the eight stars, and some might prefer to use solution 2 both for the twelve stars and the eight stars. But the case is not so clear for the latter. The means indicate that there is a sensible mean displacement of the twelve stars relatively to the eight, which is probably similar in character to the

displacement between the two exposures already noticed in the case of plate 2222. If the cause is rightly assigned then for the faint stars near the Nova it would certainly seem preferable to use solution 1, which is derived from stars more nearly of the same brightness. Or we might possibly use solution 2 corrected by the mean values found above, which would correspond to determining scale value and orientation from stars widely separated and applying a local correction to the general region.

On the whole it was determined to use solution 1—from the twelve stars—as it stands.

The places of the Nova are

	ξ_1	η_1	R.A. h m s	Decl. ° ′ ″
Solution 1	17·7168	13·5372	6 37 48·98	+30 2 38·4
Solution 2	17·7181	13·5361	6 37 49·00	+30 2 38·0

It may be mentioned that when the constants a, b, d, e are corrected for differential refraction they become

	a.	*b.*	*d.*	*e.*
Solution 1	−·00043	−·00163	+·00157	−·00073
Solution 2	−·00037	−·00170	+·00174	−·00058

The following table (V.) gives the comparison of the positions derived from both plates for those stars which have been measured on both and not given in Table IV.

TABLE V.

Ref. No.	Diameter. Pl. 2240.	2240−2222. *x.*	*y.*	Ref. No.	Diameter. Pl. 2240.	2240−2222. *x.*	*y.*
35	41	+·0009	−·0005	81	21	+·0021	+·0007
39	21	+ 2	− 15	82	18	− 23	− 7
40	39	+ 22	− 5	101	21	− 2	+ 32
41	20	− 14	− 17	108	18	− 7	− 30
42	15	− 6	− 9	109	22	+ 8	+ 20
47	23	− 5	+ 20	114	22	+ 3	− 5
51	30	− 17	+ 2	121	34	− 7	+ 4
55	19	+ 9	− 9				

In Table VI. the positions of stars deduced from plate 2222 are entered in preference to those deduced from plate 2240, since the former depend upon eight bisections and the latter on two bisections. Positions derived from plate 2240 are marked with an asterisk to the magnitude.

The stars in Father Hagen's *Chart and Catalogue for Observing Nova Geminorum* and those given in the *Harvard Circular*, No. 70, have been identified so far as they come within the Oxford range of 15′ radius from the Nova; the number, or letter, and magnitudes are entered to the corre-

sponding stars in the Oxford Catalogue; other synonyms are given at the end.

Hagen Nos. 1 to 19, 21, 23 to 26, 29, 32 to 36, 38 to 40, 42, 45, 46, and 59 are outside the limit of the 15′ radius from the Nova, and have not been measured on the Oxford plates. I have not been able to satisfactorily identify any of the stars I have measured with Nos. 57 and 58, nor after re-examination can I find a star to agree closely with either position given. Nos. 30 (10·1 mag.) and 49 (10·9) form a double star and equal Harvard *h* (9·71). Harvard *a*, *b*, *c*, *d*, *g*, *k* are outside the 15′ limit; the others are identified.

Oxford No. 167 has been specially included, as it is Barnard's reference star used for the micrometer measures with the 40-inch refractor.

The fifth column in Table VI. contains the magnitude inferred from a comparison of the Oxford diameters—the two plates being separately treated—with the magnitudes given by Harvard, Hagen, and Barnard. These magnitudes were plotted with Oxford diameters as the argument, and are practically represented by a straight line through the points. The magnitudes in the fifth column were read off from that diagram, and are only intended to be approximately correct.

TABLE VI.

Places of 126 *stars found from Plate* 2222 *and* 41 *additional stars from Plate* 2240.

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford measured diameter.	Magnitude. Inferred.	Magnitude. Hagen. Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	Deduced. N. Dec. 1900·0. ° ′ ″
1			12	12·5		14·6455	10·7932	6 36 37·93	29 48 57·6
2			15	12·1		14·6582	15·3938	6 36 38·37	30 11 57·8
3			12	12·5		14·7000	12·7556	6 36 39·25	29 58 46·3
4			18	11·7		14·7610	14·7772	6 36 40·73	30 8 52·8
5	37		29	10·5	10·4	14·8152	13·2850	6 36 41·92	30 1 24·5
6			18	11·7		14·9479	16·0553	6 36 45·09	30 15 16·1
7			18	11·7		14·9810	10·8379	6 36 45·66	29 49 10·9
8			11	12·6		15·0949	15·8957	6 36 48·50	30 14 28·2
9			7	13·2		15·1642	14·3726	6 36 50·05	30 6 51·2
10	69		18	11·7	11·5	15·2171	15·0835	6 36 51·30	30 10 24·4
11			7	13·2		15·2325	12·3740	6 36 51·53	29 56 51·6
12			9	12·8		15·2970	12·8673	6 36 53·04	29 59 19·5
13	55		20	11·5	11·1	15·6143	10·6171	6 37 0·26	29 48 4·3
14			7	13·2		15·6082	14·9459	6 37 0·34	30 9 42·9
15			20	11·5		15·7151	11·4305	6 37 2·62	29 52 8·3

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford' measured diameter.	Magnitude. Inferred.	Magnitude. Hagen.	Magnitude. Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	Deduced. N. Dec. 1900·0. ° ′ ″
16			8	13·1			15·7661	15·3371	6 37 4·01	30 11 40·2
17			20	11·5			15·9438	13·2044	6 37 7·99	30 1 0·2
18	53		22	11·2	11·1		15·9882	11·6652	6 37 8·94	29 53 18·5
19			10	12·7			15·9860	16·0686	6 37 9·13	30 15 19·4
20			8	13·1			16·0087	15·4427	6 37 9·63	30 12 11·6
21	64		18	11·7	11·4		16·0665	16·5062	6 37 11·03	30 17 30·7
22			8	13·1			16·1353	15·8863	6 37 12·59	30 14 24·7
23(1)	31	*l*	34	9·9	10·2	10·13	16·1803	13·7054	6 37 13·49	30 3 20·3
24			8	13·1			16·2904	11·8839	6 37 15·92	29 54 23·8
25			14	12·2			16·3079	10·9519	6 37 16·27	29 49 44·2
26			10	12·7			16·3504	13·1421	6 37 17·39	30 0 41·2
27	61	*r*	20	11·5	11·3	11·63	16·3557	12·6059	6 37 17·47	29 58 0·4
28			12	12·5			16·3942	12·8241	6 37 18·37	29 59 5·8
29			9	12·8			16·3932	15·2124	6 37 18·52	30 11 2·3
30			9	12·8			16·4758	13·5725	6 37 20·31	30 2 50·3
31	66		13	12·3	11·5		16·5041	14·5302	6 37 21·03	30 7 37·5
32			12	12·5			16·5121	12·2124	6 37 21·06	29 56 2·2
33			8	13·1			16·5962	16·2618	6 37 23·28	30 16 16·9
34			15	12·1			16·6679	14·6652	6 37 24·82	30 8 17·9
35	44	*n*	30	10·3	10·7	10·66	16·6754	13·3804	6 37 24·91	30 1 52·4
36			20	11·5			16·6957	12·0114	6 37 25·28	29 55 1·7
37(2)	7		30	10·3	9·9		16·7057	10·8941	6 37 25·42	29 49 26·5
38			9	12 8			16·7779	15·4014	6 37 27·41	30 11 58·6
39			13	12·3			16·8047	13·0248	6 37 27·86	30 0 5·6
40	47		26	10·7	10·9		16·8141	12·4133	6 37 28·04	29 57 2·2
41		*x*	9	12·8		12·45	16·8134	13·8450	6 37 28·13	30 4 11·7
42			5	13·5			16·8158	13·4693	6 37 28·15	30 2 19·0
43	52		23	11·1	11·0		16·8247	11·7880	6 37 28·24	29 53 54·6
44			11	12·6			16·8261	15·7081	6 37 28·55	30 13 30·6
45			17 *	13·4			16·8626	14·1116	6 37 29·29	30 5 31·6
46			10	12·7			16·8579	15·8402	6 37 29·30	30 14 10·2
47	77	*w*	16	12·0	11·7	12·23	16·8625	14·3268	6 37 29·30	30 6 36·2
48			14 *	14·0			16·8836	14·1689	6 37 29·78	30 5 48·8
49			7 *	15·0			16·9110	14·3271	6 37 30·42	30 6 36·2
50			6 *	15·2			16·9573	13·9191	6 37 31·46	30 4 33·8
										0 0 2

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford measured diameter.	Magnitude. Inferred.	Hagen.	Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	N. Dec. 1900·0. ° ′ ″
51	73	*t*	19	11·6	11·6	11·78	16·9664	14·1132	6 37 31·66	30 5 32·0
52			13	12·3			16·9748	12·2427	6 37 31·74	29 56 10·8
53	68		17	11·8	11·5		16·9739	16·1773	6 37 32·02	30 15 51·2
54			13	12·3			17·0410	12·5035	6 37 33·28	29 57 29·0
55			10	12·7			17·0557	13·0124	6 37 33·67	30 0 1·7
56			9 *	14·7			17·0946	13·5336	6 37 34·60	30 2 38·0
57			16 *	13·6			17·1448	12·5410	6 37 35·68	29 57 40·1
58			8 *	14·8			17·1613	12·5440	6 37 36·06	29 57 41·0
59	43		24	11·0	10·7		17·2180	11·5484	6 37 37·29	29 52 42·3
60			9	12·8			17 2269	11·9875	6 37 37·53	29 54 54·0
61			9 *	14·7			17·2753	13·5434	6 37 38·78	30 2 40·7
62			8	13·1			17·2907	16·2645	6 37 39·36	30 16 16·9
63			9 *	14·7			17·3154	12·5628	6 37 39·62	29 57 46·5
64			17	11·8			17·3582	15·8287	6 37 40·89	30 14 6·0
65			11	12·6			17·3752	11·4044	6 37 40·91	29 51 58·9
66			11	12·6			17·3820	15·3677	6 37 41·40	30 11 48·1
67			12	12·5			17·3844	16·1307	6 37 41·53	30 15 36·8
68			19 *	13·2			17·4723	13·7699	6 37 43·35	30 3 48·5
69			11 *	14·4			17·5784	14·1742	6 37 45·84	30 5 49·6
70			6	13·3			17·5908	12·4200	6 37 45·98	29 57 3·5
71			5 *	15·3			17·5913	13·1024	6 37 46·04	30 0 28·1
72			4 *	15·5			17·6120	12·9828	6 37 46·51	29 59 52·2
73			7 *	15·0			17·6301	12·8326	6 37 46·91	29 59 7·1
74	80	*β*	18 *	13·3		13·35	17·6380	13·2692	6 37 47·13	30 1 18·1
75			4 *	15·5			17·6614	13·7142	6 37 47·71	30 3 31·6
76			6	13·3			17·6683	14·6618	6 37 47·97	30 8 15·8
77	Nova						17·7173	13·5379	6 37 48·99	30 2 38·6
78	48	*p*	25	10·8	10·9	11·19	17·7193	14·7677	6 37 49·15	30 8 47·5
79			5 *	15·3			17·7308	13·0813	6 37 49·26	30 0 21·6
80			10 *	14 6			17·7454	13·3916	6 37 49·63	30 1 54·7
81			6	13·3			17·7878	12·6121	6 37 50·54	29 58 0·8
82	79	*α*	6	13·3		13·05	17·8161	13·8672	6 37 51·31	30 4 17·2
83	81	*γ*	16 *	13·6		13·53	17·8192	13·5128	6 37 51·34	30 2 30·9
84			9 *	14·7			17·8520	12·8930	6 37 52·04	29 59 24·9
85			5 *	15·3			17·8670	13·4114	6 37 52·44	30 2 0·4

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford measured diameter.		Magnitude. Inferred.	Hagen.	Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	N. Dec. 1900·0. ° ′ ″
86			8		13·1			17·8928	15·3043	6 37 53·22	30 11 28·3
87			9	*	14·7			17·9183	12·8027	6 37 53·57	29 58 57·8
88	51		25		10·8	11·0		17·9253	11·2580	6 37 53·57	29 51 14·4
89			6	*	15·2			17·9706	13·1547	6 37 54·81	30 0 43·3
90			13		12·3			17·9951	11·1471	6 37 55·18	29 50 41·0
91			8	*	14·8			18·0051	12·6535	6 37 55·55	29 58 12·9
92			8		13·1			18·0060	14·7797	6 37 55·78	30 8 50·7
93			5	*	15·3			18·0299	13·8081	6 37 56·24	30 3 59·3
94			9	*	14·7			18·0710	14·3579	6 37 57·24	30 6 44·1
95			9	*	14·7			18·1171	14·4097	6 37 58·31	30 6 59·6
96			9	*	14·7			18·1390	14·3376	6 37 58·81	30 6 38·0
97			11	*	14·4			18·1842	12·8489	6 37 59·71	29 59 11·3
98			13	*	14·1			18·1847	13·0967	6 37 59·75	30 0 25·6
99			11	*	14·4			18·1901	12·5512	6 37 59·81	29 57 42·0
100			11	*	14·4			18·2499	14·4758	6 38 1·39	30 7 19·3
101		*z*	8		13·1		12 95	18·3204	13·3278	6 38 2·90	30 1 34·8
102	67		18		11·7	11·5		18·3260	16·2598	6 38 3·34	30 16 14·3
103			5	*	15·3			18·3405	14·2407	6 38 3·60	30 6 8·6
104			16	*	13·6			18·3529	13·4138	6 38 3·66	30 2 0·5
105			5		13·5			18·3826	11·7564	6 38 4·18	29 53 43·3
106			10		12·7			18·4275	12·4931	6 38 5 29	29 57 24·2
107			10		12·7			18·4157	15·4065	6 38 5·32	30 11 58·2
108			5		13·5			18·4277	13·7633	6 38 5·42	30 3 45·3
109			11		12·6			18·4554	12·8191	6 38 5·97	29 59 2·0
110			16	*	13·6			18·4520	14·2097	6 38 6·04	30 5 59·2
111			5	*	15·3			18·4714	13·5990	6 38 6·42	30 2 55·9
112			7	*	15·0			18·4770	13·9158	6 38 6·58	30 4 31·0
113			6	*	15·2			18·4842	12·9924	6 38 6·65	29 59 53·9
114	75	*y*	8		13·1	11·7	12·76	18·6107	13 5907	6 38 9·64	30 2 53·3
115			12		12·5			18·6309	11·0634	6 38 9·83	29 50 15·1
116			16		12·0			18·6289	16·1998	6 38 10·34	30 15 55·9
117			17	*	13·4			18·6493	12·7503	6 38 10·44	29 58 41·1
118			10	*	14·6			18·6522	13·7664	6 38 10·61	30 3 45·9
119			12	*	14·2			18·6621	14·1938	6 38 10·89	30 5 54·1
120			11		12·6			18·6840	15·2661	6 38 11·50	30 11 15·7

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford measured diameter.	Magnitude. Inferred.	Magnitude. Hagen.	Magnitude. Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	Deduced. N. Dec. 1900·0. ° ′ ″
121	56	*q*	22	11·2	11·2	11·41	18·7368	13·0909	6 38 12·49	30 0 23·1
122			20	11·5			18·7952	16·3749	6 38 14·22	30 16 48·2
123			15	12·1			18·8243	12·5622	6 38 14·46	29 57 44·4
124			13	12·3			18·8172	16·0367	6 38 14·69	30 15 6·7
125			13	12·3			18·8994	11·9652	6 38 16·12	29 54 45·2
126	76	*s*	17	11·8	11·7	11·73	18·9057	14·6775	6 38 16·58	30 8 18·9
127			14	12·2			18·9908	14·9177	6 38 18·57	30 9 31·1
128			10	12·7			19·1165	12·7758	6 38 21·22	29 58 48·0
129	62		16	12·0	11·4		19·2276	12·2708	6 38 23·73	29 56 16·4
130	78	*u*	14	12·2	11·7	12·08	19·2654	14·0276	6 38 24·81	30 5 3·3
131			7	13·2			19·3090	10·6358	6 38 25·39	29 48 5·8
132	63		14	12·2	11·4		19·3737	13·4091	6 38 27·25	30 1 57·6
133			10	12·7			19·4074	11·1205	6 38 27·74	29 50 31·0
134	49	*h*	28	10·6	10·9	9·71	19·3959	14·5240	6 38 27·90	30 7 32·0
135	30	*h*	30	10·3	10·1		19·3969	14·4981	6 38 27·91	30 7 24·3
136(3)	41		27	10·7	10·6		19·4209	12·5410	6 38 28·23	29 57 37·1
137			11	12·6			19·5013	16·2526	6 38 30·55	30 16 10·4
138	50	*o*	22	11·2	11·0	11·01	19·5159	14·2090	6 38 30·63	30 5 57·4
139			18	11·7			19·5381	11·7121	6 38 30·83	29 53 28·3
140			13	12·3			19·6334	14·7282	6 38 33·41	30 8 32·9
141			8	13·1			19·7583	15·9519	6 38 36·46	30 14 39·8
142(4)	28	*m*	32	10·1	10·0	10·41	19·8417	12·4610	6 38 37·92	29 57 12·4
143(5)	20	*f*	41	9·2	9·4	9·14	19·8365	15·3749	6 38 38·19	30 11 46·6
144	71		14	12·2	11·6		19·8739	10·6620	6 38 38·43	29 48 12·7
145			11	12·6			19·9379	13·9164	6 38 40·34	30 4 28·8
146	54		17	11·8	11·1		19·9400	15·0680	6 38 40·54	30 10 14·3
147			10	12·7			19·9570	12·4561	6 38 40·59	29 57 10·7
148			11	12·6			19·9683	15·2567	6 38 41·23	30 11 10·9
149(6)	22		37	9·6	9·5		20·0255	11·2297	6 38 42·00	29 51 2·7
150			6	13·3			20·0914	15·4611	6 38 44·16	30 12 11·9
151			14	12·2			20·1525	12·8524	6 38 45·13	29 59 9·3
152	60		18	11·7	11·3		20·1747	15·6251	6 38 46·06	30 13 1·0
153			8	13·1			20·2612	10·7170	6 38 47·36	29 48 27·8
154			9	12·8			20·2418	16·3169	6 38 47·71	30 16 28·4
155	70		16	12·0	11·5		20·2712	12·4483	6 38 47·84	29 57 7·8

Oxford Ref. No.	Hagen No.	Harvard No.	Oxford measured diameter.	Magnitude. Inferred.	Magnitude. Hagen.	Magnitude. Harvard.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0.	Deduced. N. Dec. 1900·0.
									h m s	° ′ ″
156	65		19	11·6	11·5		20·2864	11·5035	6 38 48·05	29 52 24·4
157			8	13·1			20·3600	11·1597	6 38 49·71	29 50 41·2
158			20	11·5			20·3441	15·4441	6 38 49·95	30 12 6·4
159			14	12·2			20·4220	11·1478	6 38 51·13	29 50 37·5
160			8	13·1			20·4377	13·3508	6 38 51·81	30 1 38·3
161			13	12·3			20·5135	11·0773	6 38 53·23	29 50 16·1
162	74		12	12·5	11·7		20·5379	12·6067	6 38 54·01	29 57 54·9
163			17	11·8			20·5337	16·3609	6 38 54·47	30 16 41·0
164			9	12·8			20·5491	15·9938	6 38 54·78	30 14 50·9
165			8	13·1			20·6119	12·0151	6 38 55·64	29 54 57·3
166			13	12·3			20·7040	15·9787	6 38 58·36	30 14 46·1
167	3	*e*	46	8·6	7·9	8·82	23·8665	12·3901	6 40 10·80	29 56 42·1

Notes.

(1) No. 23 is B.D. + 30°, 1309. (2) B.D. + 29°, 1324. (3) B.D. + 30°, 1315.
(4) B.D. + 30°, 1317. (5) B.D. + 30°, 1316. (6) B.D. + 29°, 1328.
No. 68 is Barnard's No. 5: No. 74 = No. 4; No. 75 = No. 3; No. 80 = No. 2; No. 82 = No. 6; and No. 83 = No. 1.
Nos. 134 and 135 form a double star, the images coalescing with the 30^m and 35^m exposures; Harvard gives the combined light (9·71) for these two.

Four stars of the 10th magnitude or brighter with the following measured coordinates in x and y

	x	y
Camb. 3447, B.D. + 30°, 1306, Harvard *g*, Hagen 11,	14·409	13·960
„ „ B.D. + 30°, 1309, „ *l*, „ 31,	16·229	13·665
Camb. 3467, B.D. + 30°, 1316, „ *f*, „ 20,	19·883	15·335
Leiden 2803, B.D. + 30°, 1320, „ *d*, „ 15,	21·134	16·433
Nova	17·766	13·498

from the first exposure of plate 2222 lie in two lines from the Nova, and serve as two pairs of pointers for detecting the position of the Nova. A line passing through each pair to the Nova is so straight that by taking the two outside stars and calculating from the measured x of the middle one, the y for a straight line, we get the calculated $y = \cdot212 + \text{Nova}$, observed $y = \cdot167 + \text{Nova}$, $O - C = -\cdot045 = -13''{\cdot}5$; for the other the calculated is 1·845, observed 1·837, $O - C = -\cdot008 = -2''{\cdot}4$. We have frequently found them useful in ascertaining the region and for determining the orientation of the photograph. The

R.A. and Decl. of B.D. + 30°, 1306, is $6^h\ 36^m\ 31^s{\cdot}44 + 30°\ 4'\ 59''{\cdot}4$, and of B.D. + 30°, 1320 R.A. is $6^h\ 39^m\ 7^s{\cdot}30$, Decl. + 30° 17′ 13″·6. The other two stars have already been given in the catalogue under No. 23 and 143.

University Observatory, Oxford:
1903 *June* 12.

On the Position of X Geminorum. By F. A. Bellamy.

This star is on both the plates (2222 and 2240) discussed in the preceding paper, and the measures and reductions have been made in precisely the same way.

Stars No. 5, 6, 7, were measured, both exposures, on plate 2222, and corrected for the mean of the two sets of constants derived from the first and second set of exposures; stars 1 to 9 were measured on plate 2240. The positions 1 to 4, 8 and 9 depend upon direct and reverse measures on the latter plate only, 5, 6, and 7 are the means of the measures on both plates. The following are the positions (1900·0) for these few stars near *X Geminorum*:

Oxford No.	Diameter.	Inferred Mag.	Standard Coordinates. ξ_1	η_1	Deduced. R.A. 1900·0. h m s	Decl. 1900·0. ° ′ ″
1	15	13·8	24·8481	17·2664	6 40 34·57	+ 30 21 2·0
2	17	13·4	24·9145	17·4173	36·14	21 47·0
3	5	15·3	25·0267	17·4543	38·75	21 57·8
4	7	15·0	25·0565	17·3201	39·41	21 17·4
5	31	11·8	25 0722	17·4325	39·80	21 51·0
6	23	12·5	25·1959	17·8365	42·76	23 51·9
7	28	12·0	25·2112	17·6554	43·08	22 57·5
8	12	14·2	25·2426	17·8293	43·85	23 49·6
9	32	11·2	25·5909	18·0591	51 98	24 57·4

The 7th star best agrees with the approximate place of *X Geminorum* as usually given; but in the case of variable star regions which are rich with stars, especially faint ones, there is some uncertainty about the identification; but I could find no other star nearer to R.A. $6^h\ 40^m\ 43^s + 30°\ 22'{\cdot}6$ * than that given for No. 7, and stars 5, 6 and 9 of the above list agree with *e*, *g* and *d* respectively of Mr. Parkhurst's paper in *A.J.* No. 428.

University Observatory, Oxford:
1903 *June* 12.

* *Astronomical Journal*, No. 428, vol. xviii. p. 160, and No. 514, vol. xxii. p. 78.

On the Relation existing between the Light Changes and the Orbital Elements of a close Binary System, with special reference to the Figure and Density of the Variable Star RR Centauri. By Alex. W. Roberts, D.Sc.

The number of *Algol* variables, with the peculiar property of rapid and continuous variation recently discovered, brings into prominence the interesting problem of the relation of light variation to orbital movement. The refinement also and completeness which is now rightly insisted on in the observation of any cycle of light changes, due evidently to the eclipse of one star by another, makes the investigation of the problem a more hopeful matter than it would have been several years ago.

The problem is not only of considerable interest, it is also of great importance. Indeed, the determination of the figure of a close binary star solely from the character of the light changes produced by the alternating eclipse of the component bodies is one of no ordinary significance in the present progressive state of astrophysical research. It is an investigation which leads us directly to the wider problem of stellar evolution.

It is also evident that a complete solution of the problem is one of no small difficulty ; not because the equations which connect the light variation of a close binary and its orbital elements are of an indefinite character, but because the numerical quantities on which the evaluation of these equations depends are exceedingly small.

Thus the problem, even in its simplest form—that is, when it deals with the revolution of two stars practically in contact—comprises at least the determination of six unknown quantities, representing the figure, relative brightness, relative size, and relative distance of the twin stars. For the determination of these six unknown terms we have only as our argument a variation never much greater than half a magnitude.

That this meagre ebb and flow would yield data for the complete detection of the causes which have produced it, or of the conditions of figure and orbit which have given it a special character, is not to be expected. Yet, although a complete solution of the problem cannot be hoped for at present, it will be helpful, I think, to indicate the amount of certainty that we may reasonably assure ourselves of, and at the same time to state the limitations to the full exposition of the problem which we know to exist.

Put simply, the problem to be solved is this : From the light variation of any close binary system, as determined by observation, to ascertain the orbit and relative dimensions, and also the figure and density of the two stars forming the system.

At the very outset of the inquiry we are compelled to make

two or three assumptions, more or less reasonable in their nature. In the present state of our knowledge we cannot deal otherwise with the problem. Further, a conditioned investigation is more conclusive than one that is controlled by no reasonable limitations.

We assume, *first*, that the light changes of all variable stars of the *Algol* type are caused by the revolution of one star round another. Where such proof is possible the spectroscope has furnished it. It is not possible, of course, to deal spectroscopically with the light of faint stars, but when the light curve of such stars is identical in type and character with the light curve of certain bright stars that are proved to be binary, it is not unreasonable to assume that these fainter stars also are binary systems.

Then, *secondly*, we assume that all portions of the surface of a close binary system are equally luminous, and that, also, there is no outer atmospheric envelope absorbing a portion of the light from a bright interior globe. The gaseous density of nearly all *Algol* variables is urged in support of this contention.

The *third* assumption is one which is made necessary by the nature of the problem. We assume that the figures of two stars revolving round each other contiguously are prolate spheroids of revolution with two unequal axes. The problem is indeterminate if we attempt to deal with an ellipsoid of revolution with three unequal axes. The consideration of an irregular figure is, of course, out of the question: an investigation of this nature premises observations correct to within $0^{m}\cdot001$, a refinement of accuracy far beyond our present reach.

It would have been possible to deal with the problem otherwise than postulating at least two of the conditions just stated. We could have assumed figures of equilibrium such as Professor Darwin deduced from theoretical considerations alone in his classical investigation "On the Figure of Equilibrium of Rotating Masses of Fluid," and we could have enshrouded these bodies with atmospheres so constituted that they would absorb just the right amount of light to make theory agree with observation. Such a mode of reconciling theory and observation, while not lacking in suggestiveness, would certainly be wanting in finality.

On the other hand, it may be contended against the plan of inquiry adopted in the present investigation, basing our system of connecting equations on two conditions not absolutely in accord with theory, or which cannot be rigorously demonstrated as tenable, that there will be a certain inconclusiveness in the solutions arrived at.

This is to a certain extent true; the weight of the objection, however, depends on the extent to which a possible inaccurate assumption, or an uncertain condition, introduces error into our conclusions. That the acceptance of the two assumptions just stated will introduce an element of uncertainty into the investigation may be possible, but the uncertainty ranges within very narrow limits. It is of the same order as the

assumption that the Earth's orbit is circular in problems dealing with stellar parallax.

Further, without some conditioned solution the problem is insoluble. This will be made evident as the investigation proceeds.

After stating the theory of the relation of light variation to orbital movement, I purpose taking, as a special application of the various formulæ employed, the light changes of the variable star

Ch 5099 *RR Centauri*

$14^h\ 9^m\ 55^s$
$-57°\ 23'{\cdot}3$ } 1900

and from these light changes deducing the form of the orbit, and the figure and brightness of the component stars.

I. *Statement of the Equations connecting Light Variation with the Orbital Movement and Dimensions of a close Binary System.*

The typical close binary system may be considered as two stars, equal in size and brightness, spherical in figure, revolving round each other in, or nearly in, contact in an orbit whose plane is at right angles to the tangential plane. The typical light curve of such a system is represented in fig. 1.

TYPICAL LIGHT CURVE OF A CLOSE BINARY SYSTEM.

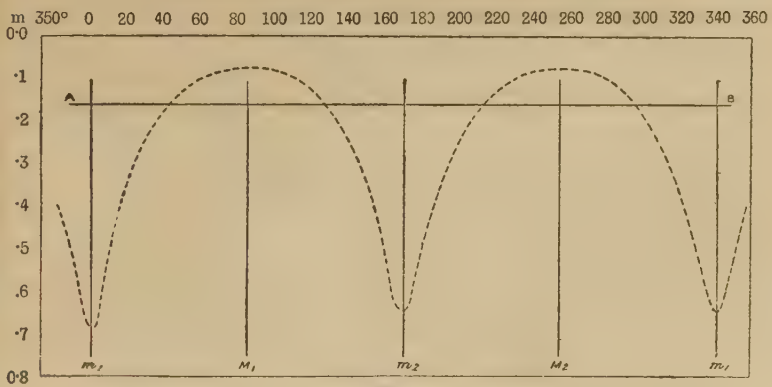


FIG. 1.

The salient features of all curves of this type are : Continuous variation, indicating that the component stars are in contact ; a considerable decrease in magnitude before the mediate line AB, fig. 1, is reached, indicating that the stars depart considerably from the spherical form ; two maxima and two minima phases, indicating that the stars are both lucid ; an equal duration of

time between the four points of the curve, m_1 M_1 m_2 M_2, indicating that the orbit of the system is practically circular.

To this typical light curve conform in a marked degree the light curves of

β Lyræ (*Astrophysical Journal*, vol. vii. p. 1).
U Pegasi (*Astrophysical Journal*, vol. viii. p. 170).
V Puppis (*Astrophysical Journal*, vol. xiii. p. 181).
X Carinæ and *RR Centauri*.

It will be evident, however, that while the light curves of these stars possess the general family likeness of the class to which they belong, they will also be marked by distinct individual differences depending on the eccentricity and inclination of the orbit in which the component stars move, as well as on the figure, brightness, and size of the stars.

TYPICAL ORBIT OF A CLOSE BINARY SYSTEM. ECCENTRICITY = 0·07.

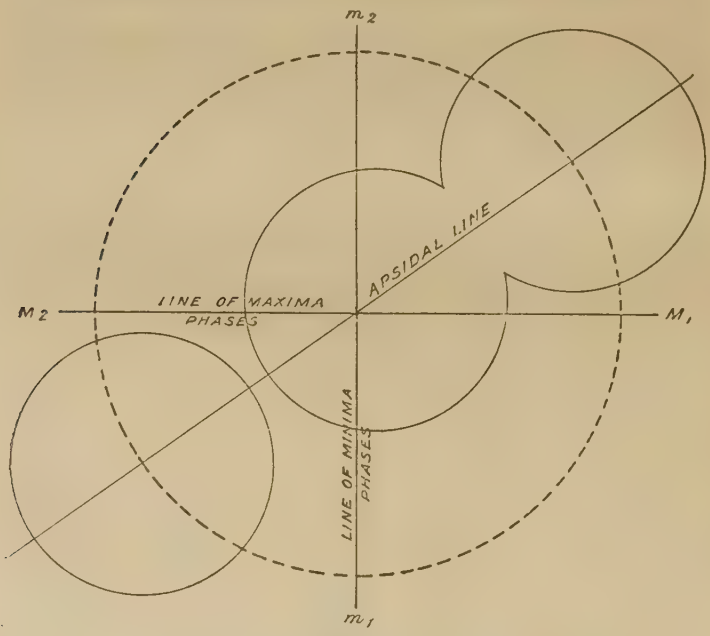


FIG. 2.

It has already been stated that *Algol* stars varying continuously cannot move in an eccentric orbit. This requires little demonstration. For let us consider as a typical system that represented in fig. 2. In this fictitious system the eccentricity

is only 0·07, but one result of this slight departure from a circular path would be a stationary period of at least one-tenth the duration of the full period of variation. An eccentricity larger than this would produce a correspondingly longer stationary period at either or both of the two maxima.

For orbits of small eccentricity a limiting relation can be thus exhibited: If P represents the full period of variation, S the duration of the stationary period at either M_1 or M_2, and e the eccentricity of the system, then

$$(1) \qquad \sqrt{e} < \frac{\pi S}{P}.$$

It is evident, therefore, that the eccentricity of a close binary system need not give us much concern. Still if the determination of the eccentricity be a matter of curiosity, there are definite data to go upon.

We have four dates as argument, viz. the instants of passage through the four points, M_1, m_2, M_2, and m_1, fig. 2.

By the fundamental law of elliptic motion the areas of the four quadrants deliminated by the lines m_1, m_2 and M_1, M_2 will be proportionate to the times of describing them. That is, the problem resolves itself into constructing an ellipse, four quadrants of which will be proportionate in area to the times taken to pass from m_1 to M_1, from M_1 to m_2, from m_2 to M_2, and from M_2 to m_1. If these four periods are equal then the orbit must be circular; if the periods are markedly unequal the orbit is eccentric, and periastron lies in the quadrant that is described in the shortest time.

The well-known formulæ of elliptical motion will in the latter case determine the eccentricity and the position of periastron, since t_1, t_2, t_3, t_4, the instants of passage through M_1, m_2, M_2, m_1, are known. Thus if we take

m°=hourly mean motion in orbit,

$t_\circ$=instant of periastron passage (taken for example in the first quadrant),

by neglecting higher powers of e, we readily get

$$(2) \qquad \begin{cases} \pi - 2(t_2 - t_1)m^\circ = 4e\{\sin(t_2 - t_\circ)m^\circ - \sin(t_1 - t_\circ)m^\circ\} \\ \pi - 2(t_3 - t_2)m^\circ = 4e\{\sin(t_3 - t_\circ)m^\circ - \sin(t_2 - t_\circ)m^\circ\} \\ \pi - 2(t_4 - t_3)m^\circ = 4e\{\sin(t_4 - t_\circ)m^\circ - \sin(t_3 - t_\circ)m^\circ\} \end{cases}$$

In the case of all close binary stars known up to the present time

$$(t_2 - t_1)m^\circ : (t_3 - t_2)m^\circ : \text{and } (t_4 - t_3)m^\circ$$

are practically equal to 90°, that is, they move, as we should expect, in circular orbits.

We now proceed to consider the equations connecting the

light variation of a close binary system with the figure, relative size, brightness, and distance of the component stars.

In any close binary system let

P=Period of variation.

E=Epoch of principal minimum phase.

T=Any time T of an observation.

ι=Inclination of orbit to plane of sight.

ac=Semi-axis-major of star$_{(1)}$.

$\lambda\, ac$=Semi-axis-major of star$_{(2)}$.

ε=Eccentricity of figure of revolution of either $S_{(1)}$ or $S_{(2)}$.

a_1=Projected or apparent semi-axis-major of $S_{(1)}$ at time T.

a_2=projected or apparent semi-axis-minor of $S_{(2)}$ at time T.

d=Apparent distance between centres of $S_{(1)}$ and $S_{(2)}$ at time T.

ϕ=Angular distance of S_1 or S_2 at time T from line of sight.

Then the following relations will be readily established :—

(3) $\phi = \left(\frac{\pm E \mp T}{P}\right) 360°$. This holds good only for a circular orbit.

(4) $d = \sqrt{1-\cos^2\phi\cos^2\iota}$

(5) $a_1 = ac\sqrt{1-\cos^2\phi\cos^2\iota\,.\,\varepsilon^2}$

(6) $a_2 = \lambda\, ac\sqrt{1-\cos^2\phi\cos^2\iota\,.\,\varepsilon^2}$

Also for simplification let

(7) $\kappa = \sqrt{1-\cos^2\phi\cos^2\iota\,.\,\varepsilon^2}$

(8) $\cos\theta_1 = \dfrac{d^2+a_1^2-a_2^2}{2a_1 d}$

(9) $\cos\theta_2 = \dfrac{d^2+a_2^2-a_1^2}{2a_2 d}$

$2\theta_1$ and $2\theta_2$ are the eccentric angles subtended by the common chord of the two auxiliary circles whose diameters are a_1 and a_2.

It follows therefore that the portion of any one of the stars eclipsed by its companion at time T will be :

(10) $$\left\{\frac{a_1\beta_1}{2}(2\theta_1-\sin 2\theta_1)+\frac{a_2\beta_2}{2}(2\theta_2-\sin 2\theta_2)\right\}$$

If now we regard S_1 as being eclipsed by S_2, the uneclipsed portion of S_1 will be

$$\pi a_1\beta_1-\left\{\frac{a_1\beta_1}{2}(2\theta_1-\sin 2\theta_1)+\frac{a_2\beta_2}{2}(2\theta_2-\sin 2\theta_2)\right\}$$

Regarding L_1 as the total light of S_1 when its axis-major is at right angles to the line of sight, it is evident that the light of the uneclipsed portion of S_1 at time T is

$$L_1 \cdot \frac{\pi a_1\beta_1 - \left\{\frac{a_1\beta_1}{2}(2\theta_1 - \sin 2\theta_1) + \frac{\lambda^2 a_1\beta_1}{2}(2\theta_2 - \sin 2\theta_2)\right\}}{\pi ac \cdot \beta c}$$

$$= L_1\left[\kappa - \frac{\kappa}{2\pi}\left\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\right\}\right]$$

To this quantity we have to add the light of the eclipsing star S_2 at time T, which is

$$(1 - L_1) \cdot \frac{a_2\,\beta_2}{\lambda ac \cdot \beta c_2}$$

$$= (1 - L_1)\,\kappa.$$

The total light of the system at any time T when S_2 eclipses S_1 is therefore

$$(1 - L_1)\,\kappa + L_1\left[\kappa - \frac{\kappa}{2\pi}\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\}\right],$$

that is

$$(11) \quad \kappa - \frac{L_1\,\kappa}{2\pi}\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\}.$$

In the case where S_1 eclipses S_2 the uneclipsed portion of S_2 will be

$$\pi a_2\beta_2 - \left\{\frac{a_1\beta_1}{2}(2\theta_1 - \sin 2\theta_1) + \frac{a_2\beta_2}{2}(2\theta_2 - \sin 2\theta_2)\right\}$$

and the light given out by this uneclipsed area will be

$$L_2 \cdot \frac{\pi a_2\beta_2 - \left\{\frac{a_1\beta_1}{2}(2\theta_1 - \sin 2\theta_1) + \frac{\lambda^2 a_1\beta_1}{2}(2\theta_2 - \sin 2\theta_2)\right\}}{\pi\lambda^2 ac \cdot \beta c}$$

$$= L_2\left[\kappa - \frac{\kappa}{2\pi\lambda^2}\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\}\right];$$

while the light from the whole system at any time T when S_1 eclipses S_2 will be

$$L_1\,\kappa + (1 - L_1)\left[\kappa - \frac{\kappa}{2\pi\lambda^2}\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\}\right]$$

that is

$$(12) \quad \kappa - \frac{(1 - L_1)\,\kappa}{2\pi\lambda^2}\{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin 2\theta_2)\}.$$

Putting now

$$(13) \quad q = \{(2\theta_1 - \sin 2\theta_1) + \lambda^2(2\theta_2 - \sin \theta_2)\}$$

and adopting the symbol L_o to indicate the light at any time T, then

when S_2 eclipses S_1

$$(14) \qquad L_o = \kappa\left(1 - \frac{L_1 q}{2\pi}\right).$$

And when S_1 eclipses S_2

$$(15) \qquad L_o = \kappa\left(1 - \frac{(1-L_1)\,q}{2\pi\lambda^2}\right).$$

Also if M_o signifies the theoretical magnitude of the system at any time T, then

$$(16) \qquad M_o = 2{\cdot}512\ (10 - \log L_o) + M,$$

where M is the maximum brightness of the system, and 2·512 the light ratio between two consecutive magnitudes.

The preceding equations indicate the series of relations that connect the changes in magnitude of any close binary system with the relative brightness, size, and magnitude of the component stars, premising the three conditions already stated.

It may be of service, in applying these equations, to collect them in an ordered sequence.

$$\phi = \left(\frac{\pm E \pm T}{P}\right) 360^\circ \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$d = \sqrt{1 - \cos^2\varphi\,\cos^2 i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$a_1 = ac\sqrt{1 - \cos^2\phi\,\cos^2 i\,.\,\varepsilon^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$a_2 = \lambda\,.\,ac\sqrt{1 - \cos^2\phi\,\cos^2 i\,\varepsilon^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

$$\cos\theta_1 = \frac{d^2 + a_1^2 - a_2^2}{2a_1 d} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

$$\cos\theta_2 = \frac{d^2 + a_2^2 - a_1^2}{2a_2 d} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Putting

$$\kappa = \sqrt{1 - \cos^2\phi\,\cos^2 i\,.\,\varepsilon^2}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

and

$$q = (2\theta_1 - \sin 2\theta_1) + \lambda^2(2\vartheta_2 - \sin 2\vartheta_2) \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

when S_2 eclipses S_1

$$L_o = \kappa\left(1 - \frac{L_1 q}{2\pi}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

and when S_1 eclipses S_2

$$L_o = \kappa\left(1 - \frac{(1-L_1)\,q}{2\pi\lambda^2}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$M_o = 2{\cdot}512\ (10 - L_o) + M \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

The method of applying these formulæ that I have found most serviceable in investigating the orbits of southern *Algol* systems has been that of successive approximations, and then correcting the final result by a system of partial differential equations. It is only possible, of course, to push the latter refinement of treatment in the case of one or two stars.

As a first approximation, for example, we might consider, as we have already done in determining the typical light curve of this class of variable, that the stars are spherical in figure, equal in size and brightness, and revolving in contact.

Taking m_0 as the mean between m_1 and m_2, and $\phi = 0°$, we have the following simple determination of the inclination of the orbit :—

$$(17)\quad \begin{cases} \log L_0 = 10 + 0{\cdot}398\,(M - m_0) \\ (2\theta - \sin 2\theta) = 2\pi\,(1 - L_0) \\ \cos i = \sqrt{1 - \cos^2\theta} \end{cases}$$

I have found it most convenient in practice, however, to determine the prolateness of the stars, and the inclination of their orbit simultaneously by a set of tables the framework of which I subjoin.

TABLE I.

	$\epsilon^2 =$									
	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
$i = 0°$	0·756	0·813	0·877	0·950	1·034	1·134	1·255	1·412	1·634	2·012
10	·539	·585	·635	·691	0·754	0·827	0·915	1·020	1·161	1·371
20	·368	·404	·443	·486	·533	·587	·649	0·723	0·816	0·943
30	·237	·265	·295	·328	·364	·404	·448	·500	·563	·641
40	·141	·162	·185	·210	·236	·264	·296	·331	·372	·420
50	·074	·089	·106	·123	·141	·161	·182	·204	·229	·257
60	·032	·042	·053	·064	·075	·086	·098	·111	·125	·140
70	·010	·015	·020	·025	·030	·036	·042	·048	·054	·061
80	·002	·003	·004	·006	·007	·009	·010	·011	·013	·015
90	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000

TABLE II.

	$\epsilon^2 =$									
	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
$i = 0°$	0·106	0·124	0·142	0·162	0·184	0·208	0·234	0·263	0·295	0·332
10	·098	·116	·135	·155	·176	·199	·224	·252	·283	·319
20	·083	·099	·116	·134	·154	·175	·198	·223	·250	·281
30	·063	·077	·091	·107	·124	·141	·160	·181	·203	·228
40	·041	·053	·065	·077	·090	·104	·118	·134	·151	·169

	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
50°	·024	·032	·040	·050	·059	·069	·079	·090	·101	·113
60	·011	·016	·022	·027	·033	·039	·045	·052	·059	·066
70	·003	·006	·009	·012	·015	·017	·020	·023	·026	·029
80	·001	·001	·002	·003	·004	·004	·005	·006	·007	·007
90	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000

Table I. gives the decrease in magnitude at minimum, as compared with the maximum brightness, for every 10° of inclination, and every 0·1 of ϵ^2; while Table II. gives the decrease in brightness for the same arguments when the eclipse is half through, that is, when $\phi = 45°$.

The application of these tables will be better exemplified by taking a concrete example.

RR Centauri is at a maximum $7^m{\cdot}37$; halfway through the eclipse it is $7^m{\cdot}53$; and the mean of the primary and secondary minima is $7^m{\cdot}80$.

That is, we take from Table I. for a decrease of $0^m{\cdot}43$, the values of ι corresponding to each tenth of ϵ^2; and from Table II. we take the values of ι and ϵ^2 for a decrease of $0^m{\cdot}16$.

These values are :—

Table III.

decrease = $0^m{\cdot}43$

when ϵ^2 =	ι =
0·0	16°·0
0·1	18·3
0·2	20·7
0·3	23·2
0·4	25·8
0·5	28·4
0·6	31·0
0·7	33·6
0·8	36·3
0·9	39·2

Table IV.

decrease = $0^m{\cdot}16$

when ϵ^2 =	ι =
0·3	7°·0
0·4	16·7
0·5	24·5
0·6	30·0
0·7	34·4
0·8	38·0
0·9	41·5

Charting down these values, as in fig. 3, we find that they coincide when

$$\iota = 32°{\cdot}3$$
$$\epsilon^2 = 0{\cdot}65$$

The other elements, ac, λ, L_1, L_2, can be now readily obtained from the formulæ.

This first approximation may be emended by varying the elements, and thus constructing equation of condition. For most stars this will be a needless and profitless refinement.

I would seek now to make a rigorous application of the foregoing to the case of the variation of *RR Centauri*, not only for the purpose of obtaining fuller knowledge of this star, but also for the purpose of ascertaining how far we can carry our investigation, and how definite and certain the results obtained are.

GRAPHICAL DETERMINATION OF ECCENTRICITY AND INCLINATION.

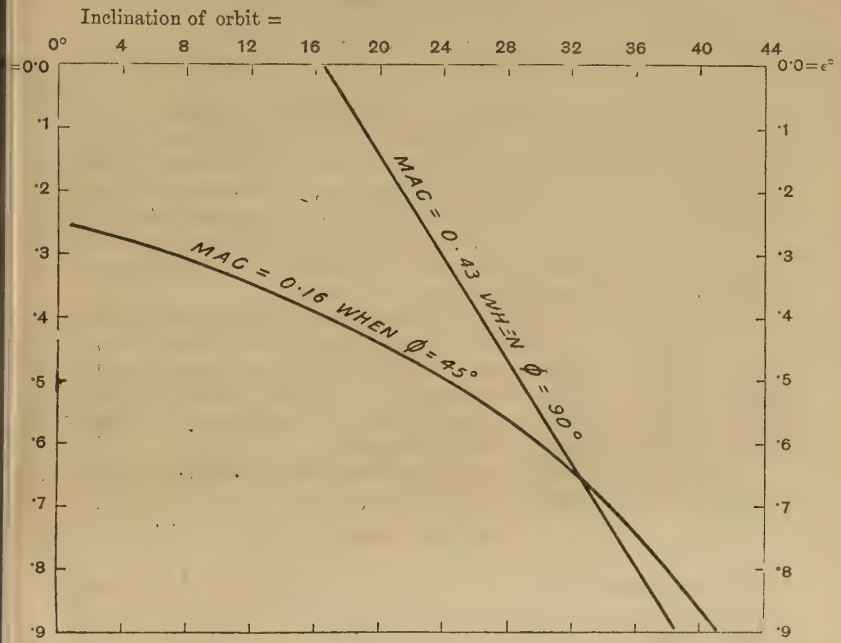


FIG. 3.

II. *Light Curve of RR Centauri as deduced from Observations taken in* 1901 *April, May, and June.*

In the months of April, May, and June 1901 a series of observations of this star were taken with the prismatic equatorial. This telescope admits of the field being rotated through any angle.

In order to eliminate the troublesome effects of position error six pointings were made each observation, the field being rotated through 60° each setting. A single observation is held to be the mean result of all the six pointings.

Five comparison stars were used in estimating the brightness of the variable, and thus for each observation thirty-six light determinations, or rather sequences, were made. Considerably over 300 observations were secured in the three months referred to, aggregating at least 12,000 light determinations.

The five comparison stars used are those dealt with in "An Investigation concerning the Position Error affecting Eye-estimates of Star Magnitudes" (*Monthly Notices*, vol. lvii. p. 485), viz.—

A. Z. C. XIV. 505	Lac. 5859	Mag. 7·24
639	5870	·40
837	5888	·57
259	5834	·62
512	—	7·87

The mode of reducing the observations employed is that described in the same paper, p. 486. This method, though laborious, ensures all the determinations of brightness being reduced to the same standard and scale. In making the observations every precaution was taken to guard against error, both irregular and systematic. Thus, in order to prevent bias influencing the determinations I closed my mind to all knowledge of the time and character of the star's variation.

I simply started observations each clear evening about seven o'clock, and went on steadily observing at intervals of twenty minutes for about eight hours. The observations were then set aside *unreduced*, and they remained unreduced until the whole series was completed. In my judgment an ephemeris of variable stars should be used as seldom as possible.

A comparison of minima determinations extending over five years gave as the period of *RR Centauri* :—

$$14^h\ 32^m\ 10^s{\cdot}76$$

With this period all the observations taken in 1901 April, May, and June were reduced to the mean curve of 1900 Jan. 1 and then grouped into twenty-nine equidistant mean places. In the formation of these mean places second differences were employed, the necessary values being furnished by a graphical mean curve.

It may be said here that while the twenty-nine mean places thus obtained represent the mean light curve of *RR Centauri*, there is manifestly an irregular, or rather regular but undetermined, secondary variation in the amplitude of the star's variation which modifies, within narrow limits, the several light curves.

The twenty-nine mean places of *RR Centauri* are as follows :—

TABLE V.

No.	Date G.M.T. 1900 d. h. m.	Observed mean mag.	Theoretical Mag. Elements III. Mag.	O−C.
1	Jan. 1 0 17	7·710	7·700	+0·010
2	47	·770	·777	−0·007
3	1 18	·810	·804	+0·006

No.	Date G.M.T. 1900 d. h. m.	Observed mean mag.	Theoretical Mag. Elements III. Mag.	O−C.
4	47	·760	·760	0·000
5	2 15	·670	·679	−0·009
6	44	·570	·586	−0·016
7	3 15	·490	·498	−0·008
8	46	·450	·432	+0·018
9	4 16	·410	·393	+0·017
10	48	·370	·376	−0·006
11	5 14	·370	·382	−0·012
12	48	·440	·418	+0·022
13	6 17	·470	·471	−0·001
14	47	·550	·547	+0·003
15	7 16	·630	·631	−0·001
16	49	·710	·723	−0·013
17	8 20	·770	·775	−0·005
18	48	·770	·764	+0·006
19	9 12	·720	·718	+0·002
20	46	·620	·620	0·000
21	10 14	·540	·537	+0·003
22	48	·460	·457	+0·003
23	11 19	·390	·405	−0·015
24	46	·370	·381	−0·011
25	12 18	·370	·377	−0·007
26	48	·390	·398	−0·008
27	13 17	·450	·442	+0·008
28	45	·510	·504	+0·006
29	1 14 19	7·610	7·603	+0·007

If the observed magnitudes given in the foregoing table are plotted down, after the usual manner, we shall have as the light curve of *RR Centauri* the distinctive curve represented in fig. 4. (Plate 21.)

It may be stated here, in connection with this figure, that the dotted line represented in the chart is the theoretical curve determined from the system of orbital elements finally arrived at in this investigation. We have, therefore, in fig. 4 (Plate 21) a graphical representation of the agreement between theory and observation.

A numerical indication of the conformity between observed and deduced magnitudes is set forth in cols. 4 and 5 of the above table.

We proceed now to consider the several steps in the determination of this theoretical curve.

III. *Determination of the Orbital Elements of RR Centauri from its observed Light Curve.*

If in fig. 4 (Plate 21) a mean curve be drawn through the observed magnitudes it will readily furnish the four dates necessary for a determination of the eccentricity of the orbit, viz.—

	d.	h.	m.
Instant of m_1 = 1900 Jan.	1	1	16 = t_1
M_1		4	55 = t_2
m_2		8	32 = t_3
Instant of M_2 =	1	12	10 = t_4

There follows therefore :

	h.	m.
$t_2 - t_1$ =	3	39
$t_3 - t_2$ =	3	37
$t_4 - t_3$ =	3	38
$t_1 - t_4$ =	3	38

These four values represent the areas of four quadrants of the orbit. But since the four areas are practically equal, it follows that the orbit of *RR Centauri* does not depart sensibly from the circular form.

We can arrive at this conclusion otherwise. It is evident from the light curve that there cannot be at either maxima a stationary period of 30^m duration. That is, from equation (1)

$$\sqrt{e} < \frac{\pi \cdot 30^m}{14^h\ 32^m}$$

$$\therefore \sqrt{e} < 0{\cdot}11$$

from which we discover that the eccentricity of *RR Centauri* cannot in any case be greater than

0·01,

and of course may be very much less.

There is conclusive evidence, therefore, that *RR Centauri* revolves in a practically circular orbit.

We have already on page 536 arrived at a first approximation of the prolateness of the figure of the stars, and of the inclination of the orbit in which they move.

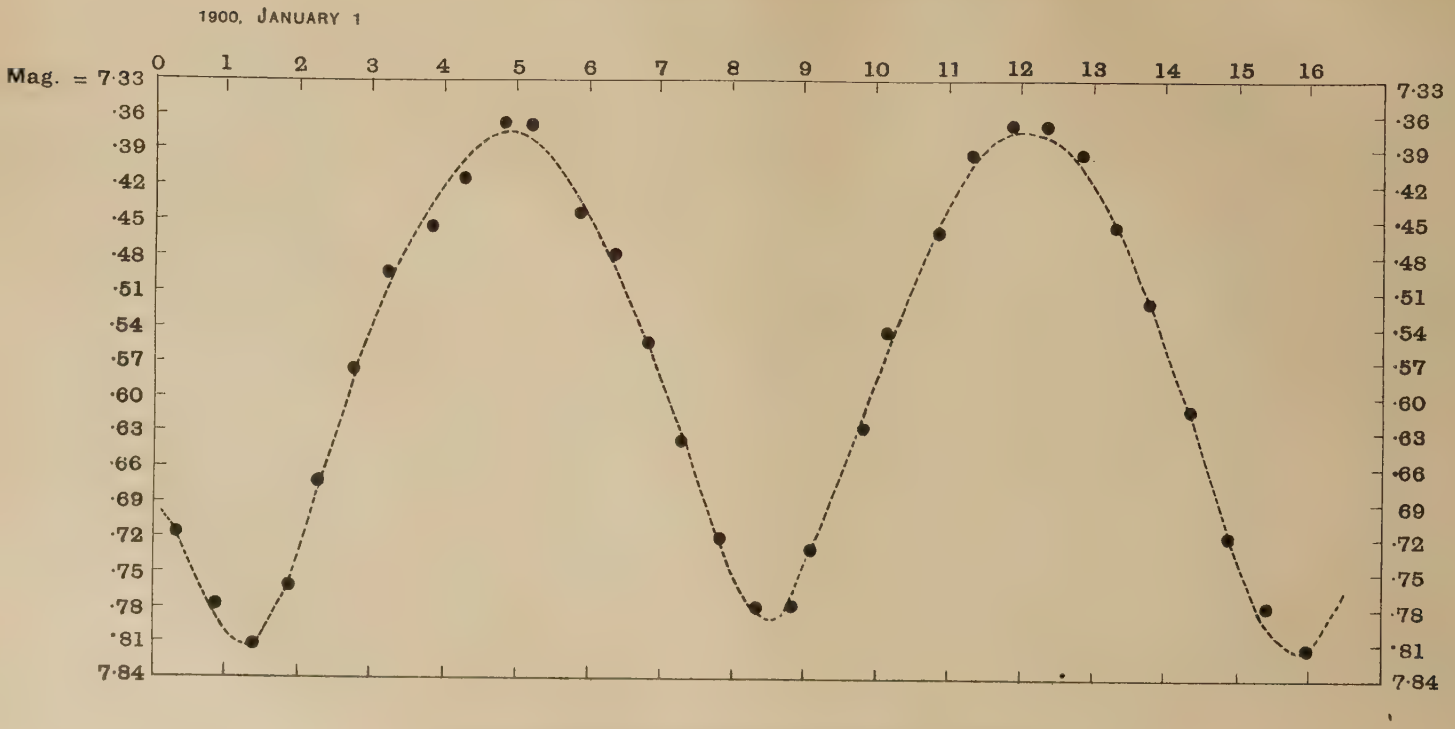
These two values are :—

$$\iota = 32{\cdot}3^\circ$$

$$\epsilon^2 = 0{\cdot}65$$

FIG. 4.

LIGHT CURVE OF RR CENTAURI



Dotted line represents Theoretical Curve obtained from Element III.

Regarding, for the first approximation, the stars as equal in size, and revolving in contact, that is—

$$ac = 0^{\circ}.5$$
$$\lambda = 1^{\circ}.0$$

we readily arrive at the three remaining values L_1, L_2, and M The only data required for this determination are the values—

$$M_1 = 7^{m}.37$$
$$M_2 = 7^{m}.37$$
$$m_2 = 7^{m}.79$$
$$m_1 = 7^{m}.81$$

These yield the following :—

$$L_1 = 0.56$$
$$L_2 = 0.44$$
$$M = 7^{m}.37$$

Grouping now all these determinations, we have the following approximate elements as a starting point :—

$P = 14^{h}\ 32^{m}\ 10^{s}.76$	Elements I.
E = 1900 January $1^{d}\ 3^{h}\ 14^{m}$	
$e = 0.00$	
$\iota = 32^{\circ}.3$	
$\varepsilon = 0.81$	
$ac_1 = 0.50$	
$ac_2 = 0.50$	
$L_1 = 0.56$	
$L_2 = 0.44$	
$M = 7^{m}.37$	

In order to compare the accuracy of the above group of elements, theoretical magnitudes were computed from them. These computed values are set forth in columns 4 and 5 of Table VI.

The theoretical magnitudes are given in column 4, and in col. 5 the divergence, O—C, between theory and observation.

TABLE VI.

Rot. No.	Date G.M.T. h. m.	Observed Mag.	Elements I. Computed Mag.	Elements I. O−C.	Elements II. Computed Mag.	Elements II. O−C.
1	0 17	7·710	7·697	+0·013	7·693	+0·017
2	47	·770	·777	− ·007	·769	+ ·001
3	1 18	·810	·807	+ ·003	·796	+ ·014
4	47	·760	·762	− ·002	·755	+ ·005
5	2 15	·670	·683	− ·013	·679	− ·009
6	44	·570	·588	− ·018	·588	− ·018
7	3 15	·490	·496	− ·006	·500	− ·010
8	46	·450	·428	+ ·022	·434	+ ·016
9	4 16	·410	·387	+ ·023	·394	+ ·016
10	48	·370	·370	·000	·377	− ·007
11	5 14	·370	·375	− ·005	·383	− ·013
12	48	·440	·412	+ ·028	·419	+ ·021
13	6 17	·470	·466	+ ·004	·470	·000
14	47	·550	·543	+ ·007	·545	+ ·005
15	7 16	·630	·632	− ·002	·630	·000
16	49	·710	·727	− ·017	·723	− ·013
17	8 20	·770	·782	− ·012	·775	− ·005
18	48	·770	·772	− ·002	·766	·004
19	9 12	·720	·724	− ·004	·720	·000
20	46	·620	·626	− ·006	·624	− 004
21	10 14	·540	·538	+ ·002	·540	·000
22	48	·460	·456	+ ·004	·460	+ ·000
23	11 19	·390	·401	− ·011	·408	− ·018
24	46	·370	·375	− ·005	·383	− ·013
25	12 18	·370	·370	·000	·379	− ·009
26	48	·390	·391	− ·001	·399	− ·009
27	13 17	·450	·434	+ ·016	·440	+ ·010
28	45	·510	·497	+ ·013	·501	+ ·009
29	14 19	7·610	7·598	+0·012	7·598	+0·012

Partial differential equations were formed on the basis of Elements I. by varying the various terms. These gave for the residuals in column 5 twenty-nine equations of condition, which

being dealt with in the usual manner gave as a second approximation—

$$\left.\begin{aligned} P &= 14^h\ 32^m\ 10^s{\cdot}76 \\ E &= 1900\ \text{January}\ 1^d\ 3^h\ 14^m \\ e &= 0{\cdot}00 \\ \iota &= 32^\circ{\cdot}71 \\ \varepsilon &= 0{\cdot}778 \\ ac_1 &= 0{\cdot}51 \\ ac_2 &= 0{\cdot}51 \\ L_1 &= 0{\cdot}542 \\ L_2 &= 0{\cdot}458 \\ M &= 7^m{\cdot}375 \end{aligned}\right\} \text{Elements II.}$$

A comparison between the magnitudes computed from this second set of elements and the observed magnitudes is given in columns 6 and 7 of Table VI.

In order to indicate the trustworthiness of this result, and the limitations we must place on its absolute definiteness, and also to show the character and scope of the investigation, I would desire to treat more fully the final corrections to this second set of elements.

The following variations on the several terms were made :—

$$\begin{aligned} \delta \cos^2 \iota &= +0{\cdot}022 \\ \delta\, \iota^2 &= +0{\cdot}055 \\ \delta\, ac_1 &= +0{\cdot}020 \\ \delta\, L_1 &= +0{\cdot}050 \\ \delta\, E &= +3^m{\cdot}0 \end{aligned}$$

These variations on the elements yielded twenty-nine equations of condition, corresponding to the twenty-nine mean places given in Tables V. and VI.

It may be of interest, as indicating the change produced in the light curve of a star by a small variation in the elements, to state these twenty-nine equations.

It will be at once evident, from a survey of the equations, that any attempted correction to elements roughly but reasonably obtained would be a futile labour unless the observations were of extreme accuracy. The measure of this accuracy is implied by the equations. The average departure from the mean of the observations used must be at least less than

$$0^m{\cdot}01$$

Indeed even with an accuracy of this degree of refinement, two of the corrections will always be indeterminate except in

special cases ; one of these exceptional cases is when the eclipse is central. The less central the eclipse, the more the uncertainty in the determination of all the elements. For this reason the star I have chosen as an exemplification of the theory stated, *RR Centauri*, presents more difficulties in the way of an accurate determination of its elements than any other variable of this class.

I have, however, selected the star out of set purpose, and for this reason, that the determination of its elements presents grave difficulties.

We may now consider the twenty-nine equations of condition.

Equations of Condition.

$+\cdot016\,\Delta\cos^2 i + \cdot019\,\Delta\epsilon^2 + \cdot016\,\Delta ac + \cdot009\,\Delta\lambda + \cdot007\,\Delta L_1 - \cdot009\,\Delta E =$						$+0\cdot017$
·022	·023	·017	·013	·011	−·006	+ ·001
·025	·025	·017	·015	·012	·000	+ ·014
·021	·022	·017	·012	·010	+·007	+ ·005
·015	·018	·016	·009	·007	·009	− ·009
·009	·013	·014	·005	·004	·009	− ·018
·005	·008	·012	·002	·002	·008	− ·010
·002	·004	·011	·001	·001	·005	+ ·016
·001	·002	·009	·001	·001	+·003	+ ·016
·000	·001	·007	+·000	+·000	·000	− ·007
·000	·001	·007	−·000	−·000	−·002	− ·013
·001	·002	·008	·001	·000	·005	+ ·021
·004	·007	·010	·002	·002	·007	+ ·000
·007	·012	·011	·003	·003	·009	+ ·005
·011	·017	·012	·005	·005	·009	+ ·000
·017	·023	·013	·008	·008	·007	− ·013
·021	·026	·013	·011	·010	−·002	− ·005
·021	·025	·013	·011	·010	+·003	+ ·004
·017	·023	·013	·008	·008	·007	+ ·000
·011	·017	·012	·005	·005	·009	− ·004
·007	·012	·011	·003	·003	·008	+ ·000
·004	·007	·010	·002	·002	·006	+ ·000
·001	·002	·008	·001	·000	·004	− ·018
·000	·001	·007	−·000	−·000	+·002	− ·013
·000	·000	·007	+·000	+·000	−·001	− ·009
·001	·001	·009	·001	·001	·003	− ·009
·003	·004	·011	·001	·001	·006	+ ·010
·005	·008	·012	·002	·002	·008	+ ·009
$+\cdot009\,\Delta\cos^2 i + \cdot013\,\Delta\epsilon^2 + \cdot014\,\Delta ac + \cdot005\,\Delta\lambda + \cdot004\,\Delta L_1 - \cdot009\,\Delta E =$						$+0\cdot012$

Multiplying each of these equations by 1,000, to reduce the

coefficients to integers, we obtain the following normal equations :—

$$
\begin{array}{rrrrrrrr}
+4092\,\Delta\cos^2\iota & +4959\,\Delta\epsilon^2 & +3599\,\Delta ac & +405\,\Delta\lambda & +208\,\Delta L_1 & +7\,\Delta E & = & +416 \\
+6160 & +4579 & +241 & +31 & +8 & & = & +368 \\
 & +4193 & +460 & +291 & -17 & & = & +227 \\
 & & +1210 & +1045 & +0 & & = & +460 \\
 & & & 911 & -7 & & = & +387 \\
 & & & & +1169 & & = & -634
\end{array}
$$

An examination of the equations of condition and also of the normal equations reveal (1) that the coefficients of $\Delta\lambda$ and ΔL_1 are so similar that it is impossible to separate them. This is a fundamental relation, and arises out of the physical conditions of the problem. Its meaning is that the same change is produced in the theoretical light curve of *RR Centauri* whether we slightly vary the size of the stars, but keep the total brightness of each star the same ; or vary their relative brightness, but keep their size the same. It is fortunate that we are able from other considerations to establish a relation between the size and the brightness of binary stars. An examination of the equations of condition also reveals (2) that the observations are refined enough to respond to the call made upon them if we except the indeterminate relation existing between $\Delta\lambda$ and ΔL_1.

In solving the preceding normal equations I have thought it better to express the unknown terms as functions of $\Delta\lambda$, leaving $\Delta\lambda$ to be determined otherwise.

The solutions are :—

$$
\begin{aligned}
\Delta\cos^2\iota &= +0\cdot284-0\cdot34\Delta\lambda \\
\Delta\epsilon^2 &= -0\cdot034+0\cdot20\Delta\lambda \\
\Delta ac_1 &= -0\cdot184+0\cdot07\Delta\lambda \\
\Delta L_1 &= +0\cdot420-1\cdot20\Delta\lambda \\
\Delta E &= -0\cdot542+0\cdot00\Delta\lambda
\end{aligned}
$$

These corrections give as the final elements of *RR Centauri* :—

Elements III.

$$
\begin{aligned}
P &= 14^h\,32^m\,10^s\cdot76 \\
E &= 1900\ \text{January}\ 1^d\,1^h\,12^m\cdot5\ (\text{G.M.T.}) \\
e &= 0\cdot000 \\
\iota &= 32^\circ\cdot320-0^\circ\cdot030\Delta\lambda \\
\epsilon &= 0\cdot776+0\cdot007\Delta\lambda \\
ac_1 &= 0\cdot506+0\cdot001\Delta\lambda \\
ac_2 &= 0\cdot506-0\cdot001\Delta\lambda \\
L_1 &= 0\cdot563-0\cdot060\Delta\lambda \\
L_2 &= 0\cdot437+0\cdot060\Delta\lambda \\
M &= 7^m\cdot375
\end{aligned}
$$

A comparison between the magnitudes computed from these elements and observation is instituted in the last two columns of Table VI.

We may also compare the relative sufficiency of each of the three elements determined by stating the average value of the O—C discordances as found from Tables V. and VI. :—

		m
Average error :	Elements I.	0·012
	Elements II.	0·010
	Elements III.	0·009

The superior accuracy of Elements III. is evident, but it is matter of doubt whether the labour involved in working out the necessary rectifications should be undertaken with stars of the narrow range of variation $0^m{\cdot}6$ of *RR Centauri*.

We have already stated that the elements finally arrived at, Elements III., would be definitive were it possible to separate ΔL_1 and $\Delta\lambda$, but from the nature of the problem this can never be done. However refined the observations, the solution must remain undeterminate for one of these two values ; indeed to discriminate between $\Delta\lambda$ and ΔL would necessitate observations correct to within $0^m{\cdot}001$.

It remains, therefore, to consider what testimony we can make use of in arriving at some idea of the value of $\Delta\lambda$.

(1) It is evident that if the two stars are in contact, they cannot be very unequal in magnitude. But from Elements III.,

$$L_1 = 0{\cdot}563 - 0{\cdot}060\Delta\lambda$$
$$L_2 = 0{\cdot}437 + 0{\cdot}060\Delta\lambda$$

that is, under no circumstances can $\Delta\lambda$ be greater than 9.

(2) If the star S_1 be very much larger than S_2, or S_2 than S_1, it is also evident that there must be a stationary period at m_1 or m_2. Such *Algol* variables are known, and they form a distinct type. We can, from geometrical considerations, place a limit to the value of the ratio λ if there is no stationary period.

In the case where the stars move in contact

$$\lambda < \frac{\sqrt{1-\epsilon^2}+\sin\iota}{\sqrt{1-\epsilon^2}-\sin\iota}.$$

In the case where the stars are not in contact

$$\lambda < 1 + \frac{\sin\iota(1+e)}{r}.$$

We find therefore a major limit for $\Delta\lambda$ by either of these methods. In the present case $\Delta\lambda$ cannot be greater than 9 ;

that is, for the four important elements we may state with confidence as a margin of error :

$$\begin{aligned} i &= 32^{\circ}\cdot320 \pm 0^{\circ}\cdot270 \\ \epsilon &= 0\cdot776 \pm 0\cdot063 \\ ac_1 &= 0\cdot506 \pm 0\cdot009 \\ ac_2 &= 0\cdot506 \pm 0\cdot009 \end{aligned}$$

There is distinct evidence of a nexus between the twin stars in the values

$$\begin{aligned} ac_1 &= 0\cdot506 \\ ac_2 &= 0\cdot506 \end{aligned}$$

These indicate that the two stars impinge on each other to the extent of

$$0\cdot012$$

the radius of the orbit being taken as unity.

In order to afford facility for an inquiry into the accordance of these determinations with what the mathematical theory of the revolution of two bodies round each other, in contact, demands, we proceed to determine the density of the system.

IV. *Density of the System RR Centauri.*

In a paper on the density of close double stars, *Astrophysical Journal*, vol. x. p. 308, I set forth certain convenient formulæ for the determination of the density of stars of the *Algol* type of variation.

Modifying these formulæ for stars ellipsoidal in figure, we have :

$$\text{Density of } S_1 = \frac{(0\cdot0046)^3}{ac_1{}^3(1-\epsilon^2)t^2} \cdot \frac{m_1}{m_1+m_2}$$

$$\text{Density of } S_2 = \frac{(0\cdot0046)^3}{ac_2{}^3(1-\epsilon^2)t^2} \cdot \frac{m_2}{m_1+m_2}$$

where

t = periodic time of star in years ;
ac_1 = semi-axis-major of S_1 ;
ac_2 = semi-axis-major of S_2 ;
m_1 = mass of S_1 ;
m_2 = mass of S_2 ;
ϵ = prolateness of figure.

Taking now $m_1 = m_2$, it follows that

$$\text{Density of } S_1 \text{ or } S_2 = 0\cdot33$$

the Sun's density being considered equal to unity.

This value is considerably higher than that found for the other southern *Algol* stars, and is in direct contradiction to the expectation that the nearer the members of a binary system are to one another, the more rare their physical constitution will probably be.

V. *General Conclusions.*

1. From the foregoing investigation it is evident that the determination of the prolateness of a close binary system is well within the province of exact astronomy. It is of course essential to such an investigation that the observations should be of the highest attainable accuracy.

2. It is also necessary to limit the area of the problem. Thus, in the present investigation we have assumed that the figure of a close binary system is that of two prolate spheroids whose major axes are in the same line.

This assumption is made imperative by the fact that we are dealing with relative comparisons of light, not absolute quantities; and thus the value of the axis of figure parallel to the axis of rotation must remain indeterminate.

Any combination of equations will only yield the projection of the system on a plane at right angles to the tangential plane, that is, on the plane of sight.

The equations we have adopted, and their application to the case of *RR Centauri*, or any other close binary star, do not lead to an erroneous result. They are rigorously true for two of the axes of figure, viz. the two which determine the section perpendicular to the axis of rotation.

3. In this same connection it may be noted that the total amount of light which reaches us from any curved surface is the same in quantity as would be received from the projection of this surface on the tangential plane. This follows naturally from the law that the total amount of light received from each unit of surface is the amount emitted by a unit of surface into the sine of the angle of inclination of that surface.

4. We find a limitation to the rigorous determination of the problem in another direction. It is impossible to discriminate between the relative size of the stars and their relative brightness when we are considering small changes in magnitude.

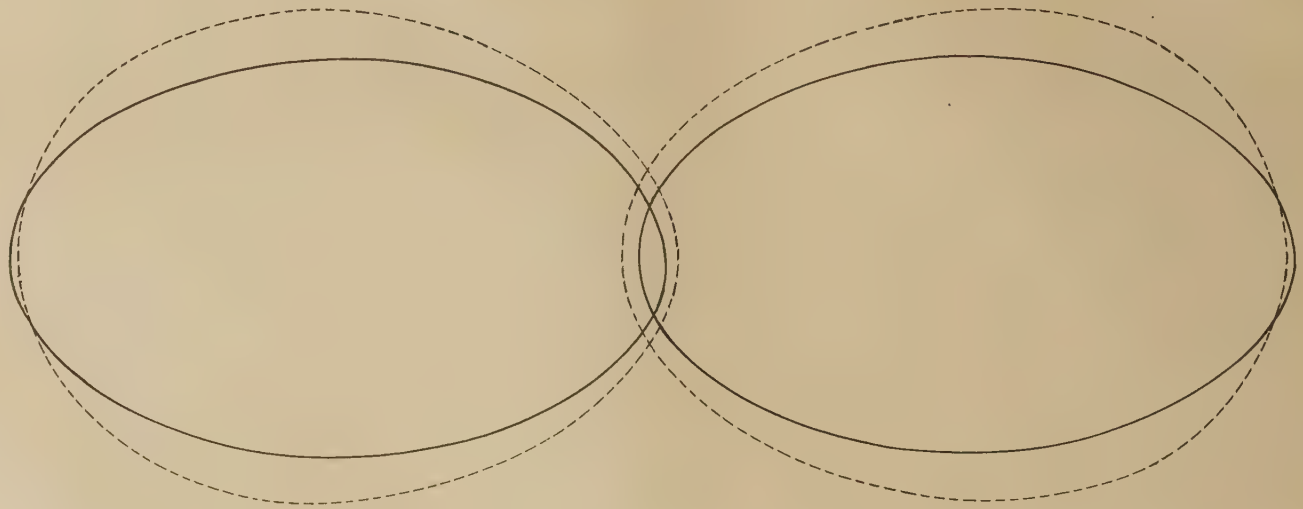
Thus the same result is obtained if we consider one star to be slightly larger than the other, but its light per unit of surface slightly less; as we do if we consider the same star to be slightly smaller, but its light intensity per unit of surface somewhat greater than that of its companion.

5. Notwithstanding these limitations it is possible to discover with some measure of completeness the elements of a close binary system from the character and form of its light curve.

This has been done in the present investigation in the case of *RR Centauri*. How far the results obtained agree with what

FIG. 5.

SYSTEM OF RR CENTAURI



Darwin's dumb-bell figure of equilibrium is indicated by dotted lines. The unbroken lines represent RR Centauri.

theory would demand is indicated in fig. 5 (Plate 22), where the system *RR Centauri* is delineated, and also the theoretical dumb-bell figure of equilibrium taken from Darwin's well-known charts, *Phil. Trans.* 1887, A, Plate 22, fig. 2.

6. The variable *RR Centauri* because of its inclination to the plane of sight, and consequent narrow range of variation, does not lend itself readily to investigations such as the present.

It is certain that more assured and more definite results would be, indeed have been, secured with stars moving practically at right angles to the tangent plane.

It seems to me reasonable, however, to take an example of more than ordinary difficulty as evidence of the possibility of pushing the observations of *Algol* variables to their legitimate issue.

Lovedale, S. Africa: 1903 *April.*

The National Argentine Observatory. By Dr. J. M. Thome.

(*Communicated by the Astronomer Royal.*)

During the sessions of the International Congress at the end of 1900 July the photographic zone comprised between the 23rd and 31st degrees of south declination, which had originally been assigned to the observatory of the province of Buenos Aires at La Plata, but which had lapsed on account of non-compliance, was by a unanimous vote transferred to the National Observatory of the Argentine Republic ; and I at once contracted with M. Gautier, the maker of the Paris telescope, for a similar instrument, with lenses by the celebrated Henry Brothers, from discs by M. Mantois ; and also a measuring machine by the same maker, a sidereal clock by Fenon, two réseaus, and various other accessories.

The boxes containing the instruments arrived in Cordoba at the beginning of 1902 January ; and, the piers having been erected previously, the telescope was installed and ready for work by the end of that month ; but, unfortunately, the Lumière plates, which I had ordered to be sent with the boxes containing the instruments, had been forgotten, and did not arrive before the end of April. These were then found to be 161 mm. square instead of 160 mm., and this being the extreme limit of the Gautier measuring machine they could not be used as catalogue plates, and, moreover, a large proportion of them were fogged. I had, also, when ordering our réseaus, ventured to change the present order of numbering the lines to one in which the numbering should begin with the lines which cross at the centre of the plate and increase both ways ; but the maker interchanged

the legends indicating the positive and negative abscissas and ordinates, and the réseau was therefore unsuitable for the charts.

After another long delay we received a gross of plates—of the proper dimensions this time—but, most unfortunately, the réseau did not conform with the focal length of our objective, being in error by more than one millimetre in 60. With this my attempts at introducing the new réseau ended, and we had recourse to the spare one in which the lines are numbered as originally, and have used it ever since. We soon found that the plates showed varying inclinations, not being true squares ; and we now always examine every plate to find the two adjacent sides which fulfil the requirements, and mark that angle before putting the plate into the holder.

The present interruptions to the work are caused almost exclusively by the Moon and bad weather. In September there was only one, in October three, and in November and December, each, two possible nights for photography. The weather has, indeed, been phenomenally bad throughout the year, there being only 173 possible nights (122 long ones) for meridian work against an average of 189 since 1890, and 270 during the first fifteen years of the existence of the Observatory. The remarkable transparency of the atmosphere has also disappeared since this region has been under irrigation, and the occasions upon which we can photograph 11th magnitude stars with 6^m exposures, owing to their inconstant light, are very rare ; we generally give from 7^m to 8^m, and even 10^m, upon dull nights. After midnight there are frequently heavy dews which interrupt the work ; and, upon some occasions, after a day of high winds and dust, we have found the lenses entirely covered with a fine dust at the end of the night, although nothing unusual was perceptible to the senses, and the nights were perfectly calm and apparently good ones.

Of the 208 plates, mainly short exposures, which we have obtained up to the beginning of April, 196 seem to fulfil the requirements, and of these a considerable number will, no doubt, show stars of the 12th magnitude owing to the uncertainty as to the proper length of exposure. The supposition is based upon the number of stars in our DM. for those regions. I think it will be found that that catalogue contains all the stars to the 10·5 magnitude upon Pickering's scale, and there are undoubtedly some in all the plate regions as faint as, or fainter than, the 11th magnitude, especially in the regions outside the Milky Way. In these regions probably everything to the $10\frac{3}{4}$ magnitude has been recorded—certainly upon the best nights.

I received from the Repsolds in September a Gill measuring machine, which is now also in use. After it had been mounted and adjusted according to Sir David's instructions I noticed a defect in the ratchet motion that carries the plate under the microscope, which threw it out of adjustment whenever the

motion was reversed by as much as the width of the double wires. I could find no one in Cordoba who was able to correct it, but finally, after a delay of some months, found an instrument maker in Buenos Aires who was able to counteract the rocking motion mechanically without altering any original part, and it now has a uniform and consistent performance in both direct and reverse motions. It affords a rapid, precise, and elegant means of determining the rectangular coordinates.

The reference stars in 23°, 24°, and 25°, from 12 to 15 upon each plate, selected from our DM., have now been observed, at least twice, in each position, circle east and circle west, of the meridian instrument. In the meanwhile I contracted with Bamberg of Berlin for a registering micrometer with clockwork attachment, which will shortly arrive, and I now propose to observe the same stars once more in position circle west. The advantage of being able to bisect an apparently stationary object, no matter what its declination or size of disc, and to record the bisection times upon a chronographic sheet, seems to me very great as compared with the present "chronographic" and "eye-and-ear" transits. I have also increased the dimensions of our meridian room, which was only 3·7 metres long, 4 wide, and 3·1 high, to 16·5 long and 3·6 high. The room now includes the collimator piers and all accessory instruments, clock, chronometers, reversing chair, &c.; and when the new micrometer arrives I confidently hope to improve upon the previous observations of these stars. The number of observations already made is nearly 26,000, and the probable error in the final positions will be about 0″·3.

The reobservations, made in former years, of all the DM. stars to the 9·3 magnitude in the region included between 22 and 37 degrees are still under revision. From a comparison with the positions given by Boss in *A. J.* Nos. 531–532, the probable error of our determinations seems to be ±0″·4.

The DM. zone included between 52° and 62° is under revision—for magnitudes principally. I have adopted Pickering's photometric scale definitely because it seems to me, after long trial, to be essentially correct; and his catalogue gives besides convenient reference belts throughout our sky. I cannot agree with Newcomb that it would have been better to continue the scale used in vols. xvi. and xvii., even supposing that I could have adhered to it consistently, which, as it was not homogeneous, would hardly have been possible. It is besides easy to reduce those magnitudes to the photometric scale, and I have given the approximate corrections in the preface to vol. xviii. p. xx. The reason why I did not estimate to tenths, or even quarter-magnitudes, while observing the stars below 10^{m} will become perfectly evident to anyone who attempts it with such means as I have; that was, moreover, supposed to be our lowest limit. In the present revision, however, I have done this in part, although it necessitates a great deal of additional work; but it will be

useful in determining what our lowest limit has really been, and as an indication that the photographic scale can be brought into good accord with the photometric, since the Cordoba and Cape scales agree, for certain plates, throughout the entire range.

The disadvantage under which I labour, in the fact that it is almost impossible to find permanent and reliable assistance here, is very great. I have had many untrustworthy men in my employ; and, in general, the foreigners who come here are true adventurers, who are not to be restrained when a prospect of better remuneration or less work offers; and their substitutes are usually drawn from the same uncertain element. This was not the case during the first period of our existence, when the remuneration was essentially upon a gold basis.

Catalogue of Right Ascensions of 76 Stars. By Robert Snow.

[The following catalogues were presented to the Society many years ago, but were not at the time considered worthy of publication. Dr. Ristenpart, who has undertaken the great work of combining all existing catalogues, on seeing the notes in *Monthly Notices*, iii. p. 99, and iv. p. 143, wrote to ask if these catalogues were still in existence. Search was made in the archives of the Society, and Mr. Snow's catalogues were found and sent (by permission of the Council) to Dr. Ristenpart. In returning them he expressed an opinion in favour of their publication; and the Council on receipt of this valuable opinion directed that the catalogues should be printed in the next following number of the *Monthly Notices*. In preparing them for publication the Editors have omitted from the first Catalogue a column of "extreme differences," and from the second a column of "greatest difference from the mean." Nos. 7 and 34 in the second Catalogue have been bracketed, as they were made with a 20-inch transit. One or two corrections have been pointed out by Dr. Ristenpart: thus the A.S.C. Nos. of 55 and 69 in the second Catalogue are given in the MS. as 1316 and 1609, and the B.A.C. number for No. 17 has been supplied. At the end of each catalogue will be found a list of corrections to the comparisons with other catalogues. It seemed desirable to give these instead of altering the text, as they are the only indication we now have of the general accuracy of the work.]

EDITORS of the *Monthly Notices*.

This catalogue was constructed with a view towards determining how nearly the results of a small instrument, used under disadvantageous circumstances, would approach to those obtained at a first-rate observatory. The instrument employed was a 20-inch transit, mounted upon the usual cast-iron stand, fixed

upon a pier of brickwork, in connexion with a chronometer showing sidereal time and beating half-seconds. The transits were, if possible, always observed over the seven wires, and when by accident, or by the intervention of clouds, the passages over any of them were lost, the remaining wires were reduced individually to the mean of all the seven by the help of a table of the equatorial distances of each wire from the mean of the whole. This table was obtained from ten complete transits of *Polaris* and twenty of δ *Ursæ Minoris*, a correction having been applied for the circular path of *Polaris*.

Numerical corrections for instrumental error have been used (viz. for the errors of inclination, collimation, and deviation in az.) in preference to attempting to destroy them by mechanical means; and in applying these corrections much trouble has been avoided by the use of Mr. Epps's tables. No correction has been applied for the inequality of the pivots.

With respect to the instrument's optical power, stars of the fourth and fifth magnitude have been considered as more agreeable to observe than brighter stars, and no difficulty has been found, on fine nights, in observing stars of the seventh magnitude. An eighth magnitude will but just allow of sufficient illumination to render the wires perceptible, and cannot be observed with any certainty; a star following β *Canis Minoris* about 42^s was found to be a good specimen of the *minimum visibile* with such an instrument in a perfectly dark field.

In observing the transits a second was taken from the chronometer when a star was approaching the first wire; and the habit of never looking again at the chronometer until the star had passed over all the wires was soon acquired. The *half-beat* of the chronometer was merely mentally acknowledged without further account being taken of it; and in this way observing with a half-seconds chronometer became a similar operation to observing with a regular transit clock. Indeed, a clock and the chronometer in question have been used on the same evening without any sense of confusion arising from the change. The stars whose places are given in the *N.A.* were used for determining the clock errors, with the exception of *Polaris* and δ *Ursæ Minoris*, whose places were taken from the Berlin Ephemeris until the appearance of the *N.A.* for 1834.

Some few additional stars have been taken for this purpose from the Greenwich Catalogue of 1112 stars [1830] during the course of last year; but it has been done but seldom. As far as possible each unknown star has been made to depend on one or more of the standard stars situated nearly in the same parallel of declination. The power used, both with and without the diagonal eyepiece, was 54; the diagonal eyepiece was laid aside whenever it was possible to do so. Of course an instrument so humble as the one in question must always be liable to great instability, which no care in using it will prevent; besides which the employment of the diagonal eyepiece is very unsatis-

factory owing to the awkward position into which the use of it forces the head in observing stars in and near the zenith, not to mention the loss of light and the additional handling of the instrument which it occasions.

The places of 67 out of the 76 stars which this catalogue consists of are to be found in the Greenwich Catalogue of 1112 stars. Five out of the 67 are in absolute agreement with the Greenwich places, and there are 41 stars whose differences from Greenwich lie between $0^s.00$ and $0^s.10$; 17 whose differences lie between $0^s.10$ and $0^s.20$, and 4 whose differences are above $0^s.20$, the widest difference being $0^s.26$. Amongst these 62 differences the + and the − signs are exactly equally divided, and the sum of the 31 + differences is $2^s.95$, and of the 31 − differences is $3^s.03$; and the difference of these two quantities is $-0^s.08$, which divided by 67 (which is the number of the stars under comparison) gives $-0^s.0012$ for the final *mean* difference of the present catalogue from that of Greenwich.

ROBERT SNOW.

September 30, 1834.

(13 *St. James's Place.*)

Ref. No.	Name of Star.	Mean R.A. 1835. h	m	s	No. of Obs. E	W	Precession. s	Diff. from Gr. 1112 Stars. s	Diff. from A.S.C. s
1	B Piscium	0	6	28·74	5	0	+3·073	...	−0·06
2	*d* Piscium	0	12	6·76	6	4	+3·077	+0·10	+0·31
3	58 Piscium	0	38	25·57	3	4	+3·111	...	−0·95
4	ϵ Piscium	0	54	23·35	5	5	+3·106	−0·06	−0·53
5	ζ' Piscium (pr^g star)	1	5	7·03	6	4	+3·112	+0·12	+0·06
6	μ Piscium	1	21	32·79	5	5	+3·111	0·00	−0·37
7	ν Piscium	1	32	51·06	3	7	+3·111	+0·10	−0·23
8	ξ' Ceti	2	4	15·76	3	4	+3·165	−0·07	+0·57
9	ν Ceti	2	27	13·34	5	5	+3·136	−0·06	−0·40
10	μ Ceti	2	36	1·79	5	5	+3·207	+0·10	−0·51
11	λ Ceti	2	50	52·78	6	4	+3·199	...	−0·62
12	*q* Orionis	4	40	53·33	5	7	+3·251	+0·04	+0·21
13	ϵ Aurigæ	4	50	8·66	7	3	+4·280	−0·09	−0·50
14	η Aurigæ	4	54	57·28	5	5	+4·182	+0·06	−0·26
15	μ Orionis	5	53	18·42	5	5	+3·295	+0·04	−0·69
16	ν Orionis	5	58	8·93	5	5	+3·421	+0·17	−0·22
17	κ Aurigæ	6	4	51·67	5	5	+3·825	+0·20	−0·54
18	811—A.S.C. (foll^g star)	6	22	40·71	6	4	+3·497	...	+0·12
19	β Canis Minoris	7	18	12·17	12	10	+3·259	−0·18	−0·66
20	ζ Cancri	8	2	44·61	5	5	+3·445	...	−0·33
21	β Cancri	8	7	33·79	6	5	+3·262	−0·01	−0·44
22	δ Hydræ	8	28	55·16	4	6	+3·185	−0·13	−0·61

Ref. No.		Name of Star.	Mean R.A. 1835. h	m	s	No. of Obs. E	W	Precession. s	Diff. from Gr. 1112 Stars. s	Diff. from A.S.C. s
23	γ	Cancri	8	33	43·62	1	4	+3·493	+0·02	−0·32
24	δ	Cancri	8	35	18·06	4	6	+3·422	−0·09	−0·44
25	ϵ	Hydræ	8	38	2·26	4	6	+3·195	−0·13	−0·43
26	ρ	Hydræ	8	39	41·37	3	7	+3·184	−0·04	−0·32
27	ζ	Hydræ	8	46	40·32	6	6	+3·183	−0·25	+0·19
28	κ	Cancri	8	58	48·34	3	7	+3·259	−0·18	+0·25
29	π	Leonis	9	51	29·43	5	5	+3·179	0·00	−0·22
30	ρ	Leonis	10	24	7·07	9	4	+3·166	+0·04	−0·02
31	l	Leonis Minoris	10	29	24·99	6	5	+3·401	+0·07	−0·21
32	n	Leonis Minoris	10	36	40·38	5	5	+3·361	+0·10	−0·66
33	o	Leonis Minoris	10	44	3·77	6	4	+3·375	−0·07	−0·20
34	σ	Leonis	11	12	37·66	6	5	+3·102	0·00	−0·48
35	ι	Leonis	11	15	19·26	6	4	+3·121	−0·14	−0·52
36	ν	Virginis	11	37	22·82	4	6	+3·086	−0·13	−0·34
37	o	Virginis	11	56	48·39	4	7	+3·071	−0·17	−0·33
38	a	Comæ Berenices	12	18	43·74	2	3	+3·011	+0·09	−0·58
39	d	Canum Venat.	12	25	53·49	2	4	+2·864	0·00	−0·05
40	ρ	Virginis	12	33	32·02	5	5	+3·030	−0·24	−0·63
41	ϵ	Virginis	12	53	57·96	6	4	+3·003	+0·06	−0·19
42	τ	Boötis	13	39	25·49	4	6	+2·883	−0·03	−0·55
43	η	Boötis	13	46	49·69	5	5	+2·859	+0·15	−0·21
44	ρ	Boötis	14	24	43·08	6	4	+2·592	−0·03	−0·02
45	ξ	Boötis	14	43	46·74	5	5	+2·753	+0·07	−0·27
46	β	Boötis	14	55	43·95	2	8	+2·261	−0·05	−0·43
47	μ	Boötis	15	18	15·44	4	6	+2·275	+0·11	−0·41
48	β	Coronæ Boreal.	15	21	1·71	5	5	+2·483	−0·08	−0·05
49	β	Serpentis	15	38	34·44	6	4	+2·757	+0·05	−0·28
50	ϵ	Serpentis	15	42	35·75	6	5	+2·972	−0·06	−0·19
51	γ	Serpentis	15	48	50·09	5	5	+2·741	+0·03	−0·08
52	h	Herculis	16	5	5·81	4	6	+2·956	...	−0·04
53	β	Herculis	16	23	7·71	5	5	+2·579	+0·03	−0·55
54	ι	Ophiuchi	16	46	12·26	5	5	+2·834	−0·08	−0·19
55	κ	Ophiuchi	16	49	51·78	5	5	+2·852	+0·10	−0·18
56	ϵ	Herculis	16	53	58·61	3	2	+2·293	+0·15	−0·34
57	δ	Herculis	17	8	15·41	2	3	+2·460	−0·01	−0·97
58	β	Ophiuchi	17	35	19·52	4	6	+2·960	−0·08	−0·30
59	S^2	Ophiuchi	17	59	31·80	8	2	+2·843	+0·03	−0·43
60	ϵ	Aquilæ	18	52	8·19	6	4	+2·723	−0·03	−0·82

Ref. No.	Name of Star.	Mean R.A. 1835. h	m	s	No. of Obs. E	W	Precession. s	Diff. from Gr. 1112 Stars. s	Diff. from A.S.C. s
61	A Aquilæ	19	11	57·44	6	4	+2·796	...	+0·18
62	*b* Aquilæ	19	17	6·14	6	4	+2·871	+0·09	−0·34
63	μ Aquilæ	19	26	1·57	7	3	+2·915	+0·10	−0·31
64	σ Aquilæ	19	31	3·07	6	4	+2·960	+0·01	+0·34
65	τ Aquilæ	19	56	4·73	5	5	+2·929	...	−0·19
66	ϵ Delphini	20	25	19·79	2	3	+2·864	0·00	−0·34
67	β Delphini	20	29	48·59	2	3	+2·803	+0·20	−0·30
68	γ Equulei	21	2	19·00	5	5	+2·912	+0·05	−0·17
69	α Equulei	21	7	34·34	5	4	+2·995	+0·26	−0·05
70	T′ Pegasi	21	30	16·12	4	2	+2·997	−0·15	−0·38
71	θ Pegasi	22	1	52·70	4	5	+3·006	−0·21	−0·05
72	ξ Pegasi	22	38	27·24	4	6	+2·975	−0·17	−0·35
73	*l* Pegasi	22	58	41·82	2	5	+3·015	...	−0·19
74	*q* Pegasi	23	20	48·90	3	5	+3·020	...	−0·44
75	O Pegasi	23	34	58·81	3	5	+3·044	...	−0·40
76	ω Piscium	23	50	50·56	6	4	+3·062	−0·04	−0·02

Note.—The following corrections should be made in the columns giving comparisons with Pond's Catalogue and the Catalogue of the Astronomical Society.—Eds. *M.N.*

		s			s
No. 1.	Difference from A.S.C.	+0·07	No. 30.	Difference from A.S.C.	−0·10
4.	„ „	−0·03	61.	„ „ „	+0·15
7.	„ „	−0·13	67.	„ „ Gr. 1112	−0·20
8	„ „	−0·43	69.	„ „ A.S.C.	−0·08
28.	„ „	+0·65	71.	„ „ Gr. 1112	−0·21

Catalogue of Right Ascensions of 125 *Stars.* By Robert Snow.

The accompanying right ascensions were observed with a transit instrument of $3\frac{1}{2}$ feet focal length and aperture $2\frac{3}{4}$ inches, made by Mr. Simms.

It has seven vertical spider's lines in its principal focus, the mean equatorial interval of which is $11^s{\cdot}81$, and has an additional vertical wire movable in a direction parallel to the rest by means of a micrometer screw, one revolution of which corresponds to three seconds of time at the equator. The instrument is mounted on stone pillars let into a thick flag-stone fixed on a brick pier worked up in cement resting on a foundation of hard flinty gravel. It is very easily reversed on its Y's,

and possesses optical power sufficient to allow of observing *Polaris* or γ *Draconis* when on the meridian with the Sun. The usual observing power is 130. The view is uninterrupted from the northern to the southern horizon. Numerical corrections have been applied to the observations for instrumental error, and tables for the purpose formed for the known and for the catalogue stars, agreeably to Mr. Epps's paper on the subject.

The instrument, from its situation and construction, has been found to be stable to a degree that is very satisfactory, but not without perceptible alterations from time to time, both in level and azimuth. The level error has of late been kept small by making occasional use of the adjusting screws, the proper numerical correction being afterwards applied to the remaining error. The collimation of the middle wire was in the first instance adjusted, and is occasionally examined, by referring it, with the assistance of the micrometer wire, to a meridian mark distant about three-quarters of a mile; and for a like purpose by night a tolerably well-defined artificial star has been formed by a bright reflecting lamp, at the same distance, shining through a circular aperture of half an inch. The error of collimation of the mean of the seven wires has been found by observing groups of known stars in reversed positions of the instrument, and comparing the clock errors so obtained.

The azimuthal adjusting screws have been rarely touched; the error, however, has never been inconveniently large, and has also had a numerical correction applied to it. This error has been found from consecutive transits of *Polaris*, and by combining *Polaris* with another known star, generally to the south of the equator; λ and δ *Ursæ Min.* have also both been similarly made use of, but much less frequently. A specimen of the printed forms made use of for registering the observations is subjoined. They have the disadvantage of rendering the observation books rather voluminous, but present great advantages in their neatness and regularity, and the ease with which they are referred to when necessity for after-discussion arises.

Polaris having been observed ten times over all the wires, the difference of the times of passing each wire from the mean of the whole was found and reduced to the equator, care having been taken to correct for the curved path of the star when at a distance from the middle wire. Thus a ready means of reducing all or any of the wires severally to the mean of the seven is obtained for stars anywhere situated, and is always resorted to in the case of a broken observation. No one wire is considered more valuable than another.

The 100 principal stars of the N.A. were employed for obtaining the clock errors, and with very few exceptions, each observation of a catalogue star is governed by a clock error derived from a mean of from three to six or eight known stars. The transit clock is a new one by Molyneux, and has the usual dead-beat escapement and mercurial pendulum.

The latitude of the place has been before stated to be 51° 15′ 58″ north, from a mean of twenty observations of stars on the prime vertical, made with a small transit instrument turned east and west. The difference of longitude between it and Mr. Wrottesley's observatory has been found by transporting between the two places a pocket mean-time chronometer by Baird belonging to Mr. Wrottesley, two box chronometers by Molyneux going mean time, and a box chronometer by Molyneux going sidereal time.

The comparisons of the chronometers with the two transit clocks were made as follows. On Thursday (May 26, 1836) at Blackheath and at Ashurst; on Saturday (May 28) at Ashurst, and at Blackheath and again at Ashurst; on Monday (May 30) at Ashurst and at Blackheath. The comparisons of the clocks and chronometers and the observations of stars determining the clock errors at both places were made by the same person. It will be sufficient to give the differences of longitude as obtained from the four chronometers and the four journeys severally, using the mean rate of each chronometer for five days.

Thursday.	m	s
No. 1055	1	12·32
No. 1119	1	12·00
No. 1115	1	12·64
Baird ...	1	13·39
Mean ...	1	12·34

Saturday Morning.	m	s
No. 1055	1	13·02
No. 1119	1	13·35
No. 1115	1	12·47
Baird ...	1	13·23
Mean ...	1	13·02

Saturday Evening.	m	s
No. 1055	1	12·79
No. 1119	1	12·68
No. 1115	1	13·26
Baird ...	1	11·87
Mean ...	1	12·63

Monday.	m	s
No. 1055	1	13·09
No. 1119	1	13·88
No. 1115	1	13·16
Baird ...	1	13·76
Mean ...	1	13·47

Final Mean.	m	s
No. 1055	1	12·34
No. 1119	1	13·02
No. 1115	1	12·63
Baird ...	1	13·47
Mean ...	1	12·86

But on Saturday the four chronometers were carried both to and from the two places; using, therefore, only the comparisons of Saturday and the proportional parts of the rates of the chronometers for that day, the four following differences of longitude are obtained:

Saturday.	m	s
No. 1055	1	12·78
No. 1119	1	13·04
No. 1115	1	12·59
Baird ...	1	12·59
Mean ...	1	12·75

On the whole the difference of longitude between the two places may be fairly stated at $1^m\ 12^s{\cdot}8$, Ashurst, west of Blackheath.

	No in A.S.C.	Star.	Mag.	R.A. Jan. 1, 1830. h m s	Precession. s	No. of Obs.	Greater than A.S.C. s	Greater than Wrottesley. s
1	19	Piscium	6·7	0 9 4·01	+3·069	2	+0·26	−0·09
2	28	*t* Piscium	6	0 16 41·46	+3·070	2	+0·41	0·00
3	35	28 Andromedæ	6	0 21 9·74	+3·136	1	+0·06	−0·40
4	47	Piscium	7	0 25 22·53	+3·092	2	+0·23	−0·01
5	82	*y* Piscium	5·6	0 40 3·41	+3·135	2	+0·19	...
6	108	ψ' Piscium	5·6	0 56 34·80	+3·191	2	−0·13	...
* [7	183	Piscium	7	1 31 38·15	+3·140	3	+0·49	−0·07]
8	212	Piscium	7	1 50 20·46	+3·194	3	+0·35	−0·01
9	398	F_1 Tauri	6·7	3 32 31·48	+3·439	1	+0·35	+0·09
10	401	F_2 Tauri	7	3 33 58·13	+3·440	1	+0·43	−0·02
11	422	Tauri	7	3 39 56·25	+3·504	2	+0·19	+0·03
12	429	Tauri	6	3 43 27·41	+3·402	2	+0·81	−0·03
13	593	Tauri	5·6	4 57 45·24	+3·541	3	+0·46	...
14	594	Tauri	6	4 57 45·50	+3·642	3	+0·21	−0·07
15	609	y^3 Orionis	6·7	5 1 56·07	+3·435	3	+0·36	−0·06
16	634	22 Aurigæ	7	5 12 37·20	+3·786	4	+0·30	+0·02
* 17	...	[B.A.C. 1657]	...	5 12 51·47	+3·057	4	...	...
18	635	*o'* Orionis	5·6	5 13 5·40	+3·055	5	+0·77	...
19	636	Aurigæ	7	5 13 41·36	+3·856	2	+0·15	−0·05
* 20	637	Aurigæ	7	5 13 41·59	+3·854	3	+0·07	−0·06
21	716	P Tauri	5·6	5 40 0·15	+3·365	3	+0·74	...
22	726	56 Orionis	5·6	5 43 36·95	+3·110	7	+0·13	...
* 23	743	Aurigæ	7	5 50 19·50	+3·765	6	+0·13	−0·04
24	748	χ^3 Orionis	5·6	5 53 23·68	+3·546	4	+0·17	...
25	755	Orionis	7	5 56 56·19	+3·440	5	+0·20	+0·01
26	762	4 Geminorum	7	6 0 11·10	+3·639	5	+0·43	−0·04
* 27	779	8 Geminorum	7	6 5 55·98	+3·663	5	+0·59	−0·17
28	792	Geminorum	7	6 14 15·43	+3·649	3	+0·22	+0·14
* 29	805	[B.D. 20°, 1442]	9·1	6 19 3·10	+3·588	1	+1·16	−0·67
30	825	25 Geminorum	7	6 30 37·96	+3·782	3	+0·68	+0·14

	No. in A.S.C.	Star.	Mag.	R.A. Jan. 1, 1830. h m s	Precession. s	No. of Obs.	Greater than A.S.C. s	Greater than Wrottesley. s
31	865	Geminorum	7	6 50 4·60	+3·446	3	+0·30	−0·01
32	889	*n* Geminorum	7	7 4 17·80	+3·671	3	+0·33	−0·04
* 33	908	58 Geminorum	7	7 13 14·90	+3·613	4	+0·85	+0·20
* [34	917	*β* Canis Min.	3	7 17 55·88	+3·259	22	+0·67	...]
35	919	*b'* Geminorum	5·6	7 18 44·45	+3·750	3	+0·57	...
36	934	Geminorum	7	7 27 5·46	+3·533	3	+0·44	−0·06
37	939	Geminorum	7	7 29 1·14	+3·853	3	...	+0·09
38	956	B Geminorum	7	7 38 23·27	+3·598	3	+0·68	+0·20]
39	967	*l* Geminorum	6·7	7 45 44·26	+3·511	3	+0·20	+0·21
40	975	Cancri	7	7 50 5·34	+3·357	3	+0·14	+0·14
41	1004	Cancri	7	8 4 26·68	+3·444	3	+0·11	+0·07
42	1080	Cancri	7	8 41 1·97	+3·412	3	+0·26	+0·19
43	1108	78 Cancri	7	8 59 30·01	+3·379	3	+0·74	+0·26
44	1119	K' Hydræ	6	9 4 2·39	+2·964	3	+0·71	+0·08
45	1128	Leonis	7	9 8 37·52	+3·265	3	+0·40	−0·01
* 46	1132	Leonis	7	9 11 15·27	+3·523	3	−1·77	...
47	1173	Leonis	7	9 33 50·74	+3·373	3	+0·62	+0·21
48	1180	20 Leonis	7	9 40 18·52	+3·377	5	+0·27	+0·25
49	1198	Leonis	6·7	9 53 19·43	+3·362	3	+0·18	+0·15
50	1202	Leonis	7	9 56 26·97	+3·272	4	+0·24	+0·17
51	1217	Leonis	7	10 5 7·53	+3·328	3	−0·04	+0·08
52	1239	*i* Sextantis	6	10 17 56·09	+3·067	3	+0·10	+0·09
53	1246	K Sextantis	6	10 20 50·73	+3·050	3	+0·51	+0·08
54	1263	50 Leonis	6·7	10 29 46·96	+3·225	3	+0·15	+0·13
55	1316	*p*4 Leonis	7	11 0 32·96	+3·066	5	+0·23	+0·12
* 56	1336	Leonis	7	11 14 35·79	+3·073	4	+0·23	+0·22
57	1348	Leonis	7	11 19 12·52	+3·065	2	+0·03	+0·01
58	1364	Virginis	7	11 29 43·21	+3·063	4	+0·49	+0·18
59	1383	Virginis	7	11 49 31·23	+3·073	5	+0·33	+0·25
60	1385	*b* Virginis	5·6	11 51 14·61	+3·072	2	+0·27	...
61	1399	*s* Virginis	7	12 1 23·49	+3·067	5	+0·29	+0·16
62	1459	Virginis	6·7	12 29 59·18	+3·079	3	−0·07	+0·13
63	1518	Virginis	7	13 5 18·18	+3·053	3	+0·12	+0·18
64	1538	*w* Virginis	5·6	13 20 6·76	+2·948	3	+0·14	...
65	1540	Virginis	7	13 20 31·59	+3·072	2	−0·06	0·00

	No. in A.S.C.		Star.	Mag.	R.A. Jan. 1, 1830. h m s	Precession. s	No. of Obs.	Greater than A.S.C. s	Greater than Wrottesley. s
* 66	1548	77	Virginis	7	13 24 32·33	+3·125	3	+0·24	+0·18
67	1551	*l*3	Virginis	6	13 26 41·08	+3·107	6	+0·13	+0·03
68	1561		Virginis	7	13 36 2·76	+3·133	10	+0·13	+0·11
69	1601		Virginis	7	13 55 21·98	+3·164	5	+0·12	+0·04
70	1638	N1	Virginis	6·7	14 18 29·49	+3·139	3	+0·40	+0·06
* 71	1651		Libræ	6·7	14 27 57·38	+3·233	5	−2·21	−0·63
* 72	1709		Libræ	7	14 56 21·10	+3·456	3	+0·41	+0·14
73	1721	*i*1	Libræ	5·6	15 2 32·90	+3·400	3	−0·04	...
74	1723	*i*2	Libræ	6·7	15 3 38·95	+3·399	3	−0·06	−0·05
75	1725	τ	Libræ	7	15 4 59·08	+3·365	3	+0·15	+0·10
* 76	1752		Libræ	7	15 20 35·86	+3·375	1	+1·26	+0·70
77	1758		Libræ	6·7	15 22 51·63	+3·426	6	+0·13	−0·01
78	1850	*c*2	Scorpii	5	16 1 50·91	+3·673	1	−0·54	...
79	1857	χ	Scorpii	6	16 4 27·21	+3·304	5	+0·02	−0·04
80	1865		Ophiuchi	6	16 7 59·28	+3·141	3	...	+0·09
* 81	1918	*g*	Draconis	5	16 39 46·06	+0·387	5	+1·09	...
82	1924		Scorpii	6·7	16 43 23·60	+3·531	3	+0·07	+0·05
83	1937		Scorpii	6·7	16 47 5·13	+3·512	3	+0·13	−0·05
84	1938	54	Herculis	5·6	16 47 54·04	+2·638	5	+0·05	...
85	1953	32	Ophiuchi	5·6	16 55 20·67	+2·740	5	+0·21	...
86	1989	70	Herculis	5·6	17 13 54·10	+2·467	5	+0·27	...
* 87	1990		Scorpii	6	17 14 32·70	+3·578	3	+0·45	+0·38
88	2004		Ophiuchi	6	17 20 9·49	+3·057	3	+0·07	−0·11
89	2223		Sagittarii	7	19 2 14·70	+3·410	1	+0·07	−0·10
90	2250	ρ2	Sagittarii	5·6	19 11 56·47	+3·496	1	+0·05	...
91	2256	χ1	Sagittarii	6	19 14 55·64	+3·654	1	+0·17	+0·24
92	2259	3	Vulpeculæ	6	19 15 53·00	+2·453	1	+0·35	−0·11
93	2310		Sagittarii	6	19 33 51·62	+3·416	4	+0·60	+0·11
94	2345		Sagittarii	7	19 49 29·27	+3·564	1	+0·27	0·00
95	2353	χ	Sagittæ	6	19 52 22·72	+2·706	2	+0·07	+0·05
* 96	2363	τ	Aquilæ	5·6	19 55 50·13	+2·929	1	+0·21	...
97	2370	*i*	Vulpeculæ	5·6	19 59 35·12	+2·573	2	−0·29	...
98	2373	ξ1	Capricorni	6·7	20 2 31·99	+3·331	1	+0·20	−0·33
99	2375	ξ2	Capricorni	6	20 2 57·13	+3·335	1	+0·21	+0·21
100	2383	*l*	Vulpeculæ	5·6	20 7 15·60	+2·460	2	+0·19	...

	No. in A.S.C.		Star.	Mag.	R.A. Jan. 1, 1830. h m s	Precession. s	No. of Obs.	Greater than A.S.C. s	Greater than Wrottesley. s
101	2431	ρ	Vulpeculæ	5·6	20 29 49·42	+2·554	4	0·00	...
102	2456	13	Delphini	5·6	20 39 23·01	+2·971	5	−0·06	...
103	2495	λ	Equulei	6	20 53 49·62	+2·957	2	−0·01	+0·08
104	2504	A	Capricorni	5·6	20 57 10·69	+3·528	3	+0·40	...
105	2529		Aquarii	7	21 12 50·82	+3·226	1	−0·01	+0·02
106	2531	y^1	Aquarii	6	21 13 49·39	+3·225	1	+0·01	+0·22
107	2535	β	Equulei	5·6	21 14 27·29	+2·974	2	+0·23	...
108	2564	5	Pegasi	5·6	21 29 48·32	+2·795	2	+0·64	...
109	2589	11	Pegasi	5·6	21 38 36·62	+3·042	3	+0·69	...
110	2599		Aquarii	7	21 43 52·85	+3·131	1	+0·30	−0·02
111	2603	B	Pegasi	5·6	21 45 19·84	+2·721	2	+0·14	...
112	2612	20	Pegasi	5·6	21 52 48·70	+2·914	3	+0·51	...
113	2642	ϕ	Piscis Austr.	5·6	22 4 10·52	+3·384	3	+0·62	...
114	2666	C	Pegasi	5·6	22 13 28·53	+2·757	3	+0·10	...
115	2682	H^2	Pegasi	5·6	22 19 15·19	+3·030	2	+0·56	...
116	2686	f	Aquarii	6	22 21 10·74	+3·222	1	+0·81	−0·52
117	2723	g^2	Aquarii	6	22 38 24·61	+3·242	1	+0·17	0·00
118	2731	σ	Pegasi	5·6	22 43 47·54	+2·999	5	+0·51	...
119	2732	K	Aquarii	6	22 44 31·02	+3·164	1	+0·26	−0·21
120	2738	1	Piscium	6	22 46 17·36	+3·067	2	−0·27	−0·25
121	2740	ρ	Pegasi	5·6	22 46 40·35	+3·010	1	+0·78	...
122	2767	m	Pegasi	5·6	23 0 56·90	+3·022	2	+0·08	...
123	2815	15	Piscium	7	23 26 47·29	+3·067	2	+0·66	+0·07
124	2841	n	Piscium	5·6	23 39 12·14	+3·076	2	+0·36	...
125	2863	ψ	Pegasi	5·6	23 49 6·69	+3·040	3	+0·08	...

Notes.

183 A.S.C. Observed with a 20-inch transit.

635 A.S.C. Too bright for a star of the 5·6 mag. The star (17) of this cat. is 2′ 5″ south of o' Orionis, and rather fainter than it.

637 A.S.C. Not noticed as faint.

743 A.S.C. Mean of two observations in 1835 ... ... s. 19·27
" " 1836 ... ... 19·57
" " 1838 ... ... 19·67

779 A.S.C. Mean of three observations in 1835 ... ... s. 55·83
Mean of two observations in 1836 ... ... 56·20

805 A.S.C. Only just visible. Observation made in consequence of Mr. Wrottesley's note upon the star. There are several faint stars in its neighbourhood.

908 A.S.C. Mean of two observations in 1836 ... ... s. 14·80
" " 1838 ... ... 15·00

917 A.S.C. β Canis Minoris. Observed with a 20-inch transit. [Same as in previous catalogue.—EDS.]

1132 A.S.C. A faint star. "Faint" also in Wrottesley's catalogue, but this star was looked for with a declination of + 25° 53′ (instead of + 26° 53′), and is not the star observed by Wrottesley.

The stars marked 7th mag. in the A.S.C. are by no means of equal brightness.

1336 A.S.C. All four observations greater than Wrottesley.

1548 A.S.C. A faint star.

1651 A.S.C. The five partial results are

s.	
57·31	One obervation was made in 1836, and the four following in 1838.
57·41	
57·34	
57·42	
57·44	
Mean ... ... 57·38	[The difference is explained by p.m. −0ˢ·06.—EDS.]
Wrottesley ... 57·74	

1709 A.S.C. Marked "too faint" at each observation. "Faint" also in Wrottesley. Mr. Baily has the words "no modern observation" opposite to it in his address.

1752 A.S.C. Excessively faint. Caught with difficulty on two wires. Date of the observation March 24, 1838. This star is called "variable" by Wrottesley.

On the day before (March 23) a star [about 3′ south of 1752 A.S.C., R.A. 15ʰ 19ᵐ 58ˢ·64 (1830)] was observed by mistake for it. [This star is ζ² Libræ = B.A.C. 5026.—EDS.]

1918 A.S.C. Greater than Pond + 0ˢ·16.

1990 A.S.C. The mean places are

s.	
32·85	observed in 1836
32·56	" 1838
32·70	" 1838

2363 A.S.C. τ Aquilæ. Mean of ten observations with a 20-inch transit, 50ˢ·08 (1830). Greatest difference from the mean 0ˢ·38. [Position from the first catalogue.—EDS.]

N.B.—The second, third, fourth, and sixth columns are copied from the A.S.C.

Note.—The following corrections should be made to Snow's comparisons with other catalogues.—EDS. *M.N.*

		s			s
No. 14.	Greater than A.S.C.	+0·12	No. 61.	Greater than A.S.C.	−0·29
20.	" " "	+0·02	64.	" " "	−0·14
24.	" " "	−0·17	84.	" " "	+0·25
32.	" " Wrottesley	+0·04	88.	" " "	−0·07
40.	" " A.S.C.	+0·35	89.	" " "	−0·07
45.	" " "	+0·34	96.	" " "	+0·24
46.	" " "	−2·77	98.	" " "	−0·20
57.	" " "	−0·03	99.	" " Wrottesley	−0·12
58.	" " "	−0·41	104.	" " A.S.C.	+0·70
59.	" " "	+0·29	125.	" " "	+0·16

Erratum in the Nautical Almanac for 1903.

(*Communicated by the Superintendent of the Nautical Almanac.*)

The assumed mean right ascension for 1903·0 of the star D.M. +16°, 1363 (p. 494 of *Nautical Almanac* 1903) is $4^s{\cdot}5$ too great. In consequence of this the times of occultation of this star on March 8, April 4, and September 15 (pp. 537 and 539) must be diminished by about 2^m in each case.

H.M. Nautical Almanac Office:
1903 *June* 11.